TRAJECTORY TRACKING CONTROL FOR FLEXIBLE-JOINT MANIPULATOR WITH TIME-VARYING UNCERTAINTIES USING BACKSTEPPING AND CHEBYSHEV NEURAL NETWORK

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Abstract

The tracking control for flexible-joint manipulator system with time-varying uncertainties is investigated in this paper. The control performance of the system is inevitably affected by the mismatched uncertainties. To tackle this issue, a novel controller that integrates backstepping and Chebyshev neural networks (CNN) is proposed. Backstepping is used to deal with the mismatched problem, and CNN are used to approximate the nonlinear functions. The adaptive law can be derived from Lyapunov stability analysis and all the signals in closed-loop system are bounded. The comparative simulation experiments validate the superior performance of the proposed method over the commonly used RBF NN.

Key Words

Flexible-joint manipulator, mismatched uncertainties, tracking control, Chebyshev neural network, backstepping, single-link

1. Introduction

Currently, robots are increasingly being used as collaborators in various settings, such as factories, hospitals, offices, and even homes [1]. Collaborative robots often incorporate flexible joints, providing advantages when encountering obstacles during operations [2]. However, several challenges arise due to complicated nonlinear terms, under-actuated and strongly coupled system, time-varying and mismatched uncertainties, and so on [3].

To address these challenges, researchers have devised various nonlinear control strategies to effectively manage flexible-joint manipulators, such as singular perturbation techniques [4], [5], backstepping control [6], [7], adaptive control [8], [9], passivity-based control [10], and intelligent control [11], [12]. The main motivation of this study is to develop a highly accurate tracking control scheme for the flexible-joint manipulator system with the time-varying uncertainties.

Backstepping is a control technique used for the design of nonlinear control systems [13], aiming to systematically transform a nonlinear control problem into a series of simpler, interconnected subsystems that can be controlled individually. However, the classic backstepping technique has a notable limitation that hinders its wide range of applications. It relies on accurate models of the system dynamics, and if there are significant deviations between the actual system behaviour and the model used for control design, the control performance may degrade. To address this issue, the function approximation technique offers significant advantages. Neural networks (NNs) are widely used to approximate any nonlinear function due to universal approximation capability. Various types of NNs have been employed in real systems, including recurrent NNs [14], interval type-2 fuzzy neural networks (IT2FNN) [15], RBF NNs (RBF) [16], and Chebyshev neural networks (CNN) [17], *etc.* The greater the number of neurons in a NN, the greater its approximation accuracy in general, but it also significantly increases the number of parameters to be estimated. To attain desirable results, it is crucial to select the appropriate parameters. CNN with the subset of Chebyshev polynomial as input has demonstrated excellent approximation capabilities [18]. In practice, CNN generally need to determine only the order of the Chebyshev polynomials, making it easier to identify the right parameters for achieving good controller performance.

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(DOI: 10.2316/J.2024.201-0446)

This study focuses on trajectory tracking control for flexible-joint manipulator system with time-varying mismatched uncertainties. A novel trajectory tracking controller is proposed by employing a combination of backstepping and CNN techniques. The use of Lyapunov stability theory enables the derivation of an adaptive law, ensuring that all signals in the closed-loop system are UUB. The proposed controller requires fewer control parameters while achieving superior control outcomes, compared to the widely used RBF NN. The main contributions of this paper are as follows:

(1) A novel trajectory tracking controller is proposed for flexible-joint manipulator system with time-varying mismatched uncertainties by integrating backstepping and CNN.
(2) The proposed controller requires fewer control parameters while achieving superior control outcomes.

The paper is organised as follows: In Section 2, the problem statement and preliminaries are presented. The proposed controller is introduced in Section 3. In Section 4, the effectiveness and superior performances of the proposed controller are verified through simulation results. The summary of the results obtained is given in Section 5.

2. Problem Statement and Preliminaries

2.1 Problem Statement

In this paper, consider the single-link flexible-joint manipulator described in [16]. Assuming that the link is rigid and ignoring the viscous damping is, its dynamic equations are expressed as follows:

$$\begin{aligned} D\ddot{q}_l + mgl\sin(q_l) + k(q_l - q_r) = 0 \\ J\ddot{q}_r - k(q_l - q_r) = \tau \end{aligned} \tag{1}$$

where $q_l \in R$ denotes the link position, $q_r \in R$ denotes the motor position. All the mentioned parameters, including the link inertia D, motor inertia J, link mass m, stiffness k, centre of mass l, and gravity constant g, possess positive values. τ is the control torque. To facilitate the design description of the problem, the state space variables $x_1 = q_l$, $x_2 = \dot{q}_l$, $x_3 = q_r$, and $x_4 = \dot{q}_r$ are defined. Due to the time-varying, mismatched uncertainty of flexible joint manipulator, we are unable to obtain a precise model. Considering the actual connection of flexible joint manipulator, the model can be simplified by treating the flexible joint manipulator as a combination of two subsystems as referred in [19]. The dynamic equations are described as follows:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 + E_1(x) \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = E_2(x) + \alpha\tau \end{cases} \tag{2}$$

where $x = [\, x_1 \; x_2 \; x_3 \; x_4 \,]^T$is the state vector, $E_1(x) = -x_3 - \frac{\mathrm{mgl}}{D}\sin x_1 - \frac{k}{D}(x_1 - x_3)$, $E_2(x) = \frac{k}{J}(x_1 - x_3)$, $\alpha = \frac{1}{J}$.

Assumption 1: Suppose that $E_1(x)$ and $E_2(x)$ are uncertain, and the bounds on their variation are uncertain.

Assumption 2: Suppsed α is an unknown constant, and is its positive lower bound.

Assumption 3: The system states are all accessible.

Our objective is to propose a trajectory tracking controller to make that q_l can track a continuous desired trajectory q_{ld} while all signals in the closed-loop feedback system are UUB. Since $E_1(x)$ and $E_2(x)$ are uncertain, in the following, a novel trajectory tracking controller is designed by using backstepping and CNN approximation.

2.2 Preliminaries

In this paper, CNN are utilised to approximate unknown nonlinear functions. CNN with the subset of Chebyshev polynomial as input is a function-link NN [20]. Chebyshev orthogonal polynomials can be expressed as follows:

$$\begin{cases} T_0(x) = 1 \\ T_{i+1}(x) = 2xT_i(x) - T_{i-1}(x) \end{cases} \tag{3}$$

where $x \in R$ and $T_1(x)$ comes with many forms as x, $2x$, $2x-1$, and $2x+1$. Here, we choose $T_1(x) = x$.

Consider a vector $x = (x_1, \ldots, x_m)^T \in R^m$, the Chebyshev polynomials can be expressed as:

$$\varphi(x) = (1, T_1(x_1), \ldots, T_n(x_1), \ldots, T_1(x_m), \ldots, T_n(x_m))^T \tag{4}$$

where n is the order of Chebyshev polynomials.

Due to the excellent approximation capabilities, $F(x) \in R^n$ can be expressed as:

$$F(x) = W^T\varphi(x) + \varpi \tag{5}$$

where W is the optimal weight matrix and ϖ is the bounded approximation error.

3. Controller Design

In this sector, a novel trajectory tracking controller is proposed by using backstepping and CNN approximation.

3.1 Backstepping Controller

Backstepping method is a commonly used and effective control method. In the controller design process, all quantities with "ˆ" denote estimates and all quantities with "∼" denote estimation errors. The steps for designing a backstepping controller for (2) are as follows.

Step 1: Define $e_1 = x_1 - q_{\mathrm{ld}}$, q_{ld} is the desired trajectory. The time derivative of e_1 is

$$\dot{e}_1 = \dot{x}_1 - \dot{q}_{\mathrm{ld}} = x_2 - \dot{q}_{\mathrm{ld}} \tag{6}$$

Define $e_2 = x_2 - x_{2d}$, where x_{2d} is a virtual control, x_{2d} is proposed as follows:

$$x_{2d} = \dot{q}_{\mathrm{ld}} - k_1 e_1 \tag{7}$$

where $k_1 > 0$. Equation (6) can be written as:

$$\dot{e}_1 = e_2 + x_{2d} - \dot{q}_{\mathrm{ld}} = -k_1 e_1 + e_2 \tag{8}$$

Define the Lyapunov function:

$$V_1 = \frac{1}{2} e_1^2 \tag{9}$$

Taking the time derivative of V_1, produce:

$$\dot{V}_1 = -k e_1^2 + e_1 e_2 \tag{10}$$

If $e_2 = 0$, then $\dot{V}_1 \le 0$.

Step 2: Taking the time derivative of e_2, produce:

$$\dot{e}_2 = \dot{x}_2 - \dot{x}_{2d} = x_3 + E_1(x) - \dot{x}_{2d} \tag{11}$$

Define $e_3 = x_3 - x_{3d}$, where x_{3d} can be selected to stabilise (11) with:

$$x_{3d} = -\widehat{E}_1(x) + \dot{x}_{2d} - k_2 e_2 - e_1 \tag{12}$$

where $k_2 > 0$, $\widehat{E}_1(x)$ is the estimate of $E_1(x)$.

Take the derivative of x_{2d}:

$$\dot{x}_{2d} = \ddot{q}_{\mathrm{ld}} - k_1 \dot{e}_1 = \ddot{q}_{\mathrm{ld}} - k_1(x_2 - \dot{q}_{\mathrm{ld}}) \tag{13}$$

The time derivative of e_2 along (11) and (12) is given by:

$$\begin{aligned} \dot{e}_2 &= e_3 + x_{3d} + E_1(x) - \dot{x}_{2d} \\ &= E_1(x) - \widehat{E}_1(x) - k_2 e_2 + e_3 - e_1 \end{aligned} \tag{14}$$

Define the following Lyapunov function:

$$V_2 = \frac{1}{2} e_1^2 + \frac{1}{2} e_2^2 \tag{15}$$

Taking the derivative of V_2, produce:

$$\dot{V}_2 = -k_1 e_1^2 - k_2 e_2^2 + (E_1(x) - \widehat{E}_1(x)) e_2 + e_2 e_3 \tag{16}$$

If $e_3 = 0$ and $E_1(x) - \widehat{E}_1(x) = 0$, then $\dot{V}_2 \le 0$.

Step 3: Taking the time derivative of e_3, produce:

$$\dot{e}_3 = \dot{x}_3 - \dot{x}_{3d} = x_4 - \dot{x}_{3d} \tag{17}$$

$\dot{x}_{3d}$ along (11)–(14) is given by:

$$\begin{aligned} \dot{x}_{3d} &= -\dot{\widehat{E}}_1(x) + \ddot{x}_{2d} - k_2 \dot{e}_2 - \dot{e}_1 \\ &= -\dot{\widehat{E}}_1(x) + \dddot{q}_{\mathrm{ld}} - k_1(x_3 + E_1(x) - \ddot{q}_{\mathrm{ld}}) \\ &\quad -k_2(x_3 + E_1(x) - \dot{x}_{2d}) - x_2 + \dot{q}_{\mathrm{ld}} \end{aligned} \tag{18}$$

$\dot{x}_{3d}$ can be expressed as$\dot{x}_{3d} = \Delta_1 - \Delta_2$, where $\Delta_1 = \dddot{q}_{\mathrm{ld}} - k_1(x_3 - \ddot{q}_{\mathrm{ld}}) - k_2(x_3 - \dot{x}_{2d}) - x_2 + \dot{q}_{\mathrm{ld}}$ is the known part, and $\Delta_2 = \dot{\widehat{E}}_1(x) + (k_1 + k_2) E_1(x)$ is the unknown part.

Define $e_4 = x_4 - x_{4d}$, where x_{4d} can be selected as:

$$x_{4d} = \Delta_1 - \widehat{\Delta}_2 - k_3 e_3 - e_2 \tag{19}$$

Substituting (19) into (17), $\dot{e}_3$ can be expressed as:

$$\dot{e}_3 = x_4 - \dot{x}_{3d} = -k_3 e_3 - e_2 + e_4 + \Delta_2 - \widehat{\Delta}_2 \tag{20}$$

Define the following Lyapunov function:

$$V_3 = \frac{1}{2} \sum_{i=1}^{3} e_i^2 \tag{21}$$

Taking the time derivative of V_3, produce:

$$\begin{aligned} \dot{V}_3 &= -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 + (E_1(x) - \widehat{E}_1(x)) e_2 \\ &\quad +(\Delta_2 - \widehat{\Delta}_2) e_3 + e_3 e_4 \end{aligned} \tag{22}$$

If $e_4 = 0$, $\Delta_2 - \widehat{\Delta}_2 = 0$ and $E_1(x) - \widehat{E}_1(x) = 0$, then $\dot{V}_3 \le 0$.

Step 4: Taking the time derivative of e_4, produce:

$$\dot{e}_4 = \dot{x}_4 - \dot{x}_{4d} = E_2(x) + \alpha\tau - \dot{x}_{4d} \tag{23}$$

$\dot{x}_{4d}$ along (17) and (19) is given by:

$$\begin{aligned} \dot{x}_{4d} &= \ddddot{q}_{\mathrm{ld}} - k_1(\dot{x}_3 - \dddot{q}_{\mathrm{ld}}) - k_2(\dot{x}_3 - \dot{x}_{2d}) + \ddot{q}_{\mathrm{ld}} - \dot{x}_2 \\ &\quad -\dot{\widehat{\Delta}}_2 - k_3(x_4 - \ddot{q}_{\mathrm{ld}} + k_1(x_3 - \ddot{q}_{\mathrm{ld}}) + k_2(x_3 - \dot{x}_{2d}) \\ &\quad -\dot{q}_{\mathrm{ld}} + x_2 + \Delta_2) - (\dot{x}_2 - \dot{x}_{2d}) \end{aligned} \tag{24}$$

$\dot{x}_{4d}$ can be expressed as $\dot{x}_{4d} = \varepsilon_1 + \varepsilon_2$, where$\varepsilon_1 = \ddddot{q}_{\mathrm{ld}} - k_2(x_4 - \dddot{q}_{\mathrm{ld}} + k_1(x_3 - \ddot{q}_{\mathrm{ld}})) + \ddot{q}_{\mathrm{ld}} - x_3 - k_3(x_4 - \Delta_1) - (x_3 - \dot{x}_{2d}) - k_1(\dot{x}_3 - \dddot{q}_{\mathrm{ld}})$ is the known part, and $\varepsilon_2 = -(k_1 k_2 + 2) E_1(x) - \dot{\widehat{\Delta}}_2 - k_3 \Delta_2$ is the unknown part.

Define $\overline{E}_2(x) = E_2(x) - \varepsilon_2$, and (23) can be written as:

$$\begin{aligned} \dot{e}_4 &= \overline{E}_2(x) + \varepsilon_2 + \alpha\tau - \dot{x}_{4d} \\ &= \overline{E}_2(x) - \dot{x}_{4d} + (\alpha - \widehat{\alpha})\tau + \widehat{\alpha}\tau \end{aligned} \tag{25}$$

where $\widehat{\alpha}$ is the estimated value of α.

The control torque τ is proposed as:

$$\tau = \frac{1}{\widehat{\alpha}}(-\widehat{\overline{E}}_2(x) + \varepsilon_1 - k_4 e_4 - e_3) \tag{26}$$

where $k_4 > 0$ and $\widehat{\overline{E}}_2(x)$ is the estimated value of $\overline{E}_2(x)$.

Substituting (26) into (25), $\dot{e}_4$ can be expressed as:

$$\dot{e}_4 = (\overline{E}_2(x) - \widehat{\overline{E}}_2(x)) + (\alpha - \widehat{\alpha})\tau - k_4 e_4 - e_3 \tag{27}$$

Define the following Lyapunov function:

$$V_4 = \frac{1}{2} \sum_{i=1}^{4} e_i^2 \tag{28}$$

Taking the time derivative of V_4, produce:

$$\begin{aligned} \dot{V}_4 &= -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 - k_4 e_4^2 + (\alpha - \widehat{\alpha})\tau e_4 \\ &\quad +(E_1(x) - \widehat{E}_1(x)) e_2 + (\Delta_2 - \widehat{\Delta}_2) e_3 \\ &\quad +(\overline{E}_2(x) - \widehat{\overline{E}}_2(x)) e_4 \end{aligned} \tag{29}$$

If $\alpha = \widehat{\alpha}$, $\overline{E}_2(x) = \widehat{\overline{E}}_2(x)$, $\Delta_2 = \widehat{\Delta}_2$ and $E_1(x) = \widehat{E}_1(x)$, then $\dot{V}_4 \le 0$.

Here, the CNN are utilised to approximate unknown $E_1(x)$, Δ_2, and $\overline{E}_2(x)$. Then, the following expressions exist:

$$
\begin{aligned}
E_1(x) &= W_1^T\varphi_1(x) + \varpi_1 \\
\Delta_2 &= W_2^T\varphi_2(x) + \varpi_2 \\
\overline{E}_2(x) &= W_3^T\varphi_3(x) + \varpi_3
\end{aligned} \tag{30}
$$

The estimated quantities of $E_1(x)$, Δ_2, and $\overline{E}_2(x)$ can be expressed as:

$$
\begin{aligned}
\widehat{E}_1(x) &= \widehat{W}_1^T\varphi_1(x) \\
\widehat{\Delta}_2 &= \widehat{W}_2^T\varphi_2(x) \\
\widehat{\overline{E}}_2(x) &= \widehat{W}_3^T\varphi_3(x)
\end{aligned} \tag{31}
$$

where $\widehat{W}_1$, $\widehat{W}_2$, and $\widehat{W}_3$ are the estimates of the optimal weights.

3.2 Adaptive Law Design

We design adaptive laws for$\widehat{\alpha}$, $\widehat{W}_1$, $\widehat{W}_2$, and $\widehat{W}_3$ through stability analysis. Define the Lyapunov function as:

$$
V = \frac{1}{2}\sum_{i=1}^{4} e_i^2 + \frac{1}{2}\mathrm{tr}(\widetilde{\Omega}^T\Xi^{-1}\widetilde{\Omega}) + \frac{1}{2}\eta\widetilde{\alpha}^2 \tag{32}
$$

where $\widetilde{\Omega} = \Omega - \widehat{\Omega}$, $\Omega = \begin{bmatrix} 0 & & & \\ & W_1 & & \\ & & W_2 & \\ & & & W_3 \end{bmatrix}$, $\|\Omega\|_F \le \Omega_M$,

$\widehat{\Omega} = \begin{bmatrix} 0 & & & \\ & \widehat{W}_1 & & \\ & & \widehat{W}_2 & \\ & & & \widehat{W}_3 \end{bmatrix}$, $\Xi = \begin{bmatrix} 0 & & & \\ & \Gamma_1 & & \\ & & \Gamma_2 & \\ & & & \Gamma_3 \end{bmatrix}$, $\eta > 0$.

We design the adaptive law as follows:

$$
\dot{\widehat{\Omega}} = \Xi\Phi\xi^T - n\Xi\,\|\xi\|\,\widehat{\Omega} \tag{33}
$$

where, $\Phi = \begin{bmatrix} 0 & \varphi_1(x) & \varphi_2(x) & \varphi_3(x) \end{bmatrix}$, $n > 0$, $\widehat{\alpha}(0) \ge> 0$, $\xi = [\, e_1 \;\; e_2 \;\; e_3 \;\; e_4 \,]^T$.

Using (29) and (32), the derivative of V is:

$$
\begin{aligned}
\dot{V} \;=\; & -k_1e_1^2 - k_2e_2^2 - k_3e_3^2 - k_4e_4^2 + (\widetilde{W}_1^T\varphi_1(x) + \varpi_1)e_2 \\
& +(\widetilde{W}_2^T\varphi_2(x) + \varpi_2)e_3 + (\widetilde{W}_3^T\varphi_3(x) + \varpi_3)e_4 \\
& +tr(\widetilde{\Omega}^T\Xi^{-1}\dot{\widetilde{\Omega}}) + \widetilde{\alpha}e_4\tau + \eta\widetilde{\alpha}\dot{\widetilde{\alpha}}
\end{aligned} \tag{36}
$$

Consider $K = [\, k_1 \;\; k_2 \;\; k_3 \;\; k_4 \,]^T$, $\varpi = [\, 0 \;\; \varpi_1 \;\; \varpi_2 \;\; \varpi_3 \,]$, $\dot{V}$can be expressed as:

$$
\begin{aligned}
\dot{V} \;=\; & -\xi^TK\xi + \xi^T\varpi + tr(\widetilde{\Omega}^T\Xi^{-1}\dot{\widetilde{\Omega}} + \widetilde{\Omega}^T\Phi\xi^T) \\
& +\widetilde{\alpha}e_4\tau + \eta\widetilde{\alpha}\dot{\widetilde{\alpha}}
\end{aligned} \tag{37}
$$

Since $\dot{\widetilde{\Omega}} = -\dot{\widehat{\Omega}}$, $\dot{\widetilde{\alpha}} = -\dot{\widehat{\alpha}}$, using (33), produce

$$
\dot{V} = -\xi^TK\xi + \xi^T\varpi + n\,\|\xi\|\,\mathrm{tr}(\widetilde{\Omega}^T(\Omega - \widetilde{\Omega})) + \beta \tag{38}
$$

where $\beta = \widetilde{\alpha}e_4\tau - \eta\widetilde{\alpha}\dot{\widehat{\alpha}}$.

We design the adaptive law of α as:

$$
\dot{\widehat{\alpha}} = \begin{cases} \eta^{-1}e_4\tau & e_4\tau > 0 \\ \eta^{-1}e_4\tau & e_4\tau \le 0 \;\; \widehat{\alpha} > \\ \eta^{-1} & e_4\tau \le 0 \;\; \widehat{\alpha} \le \end{cases} \tag{39}
$$

where the initial value $\widehat{\alpha}(0) \ge$.

Substituting (39) into β, it can be found that:

when $e_4\tau > 0$, $\beta = 0$; when $e_4\tau \le 0$, $\widehat{\alpha} >$, $\beta = 0$; when $e_4\tau \le 0$, $\widehat{\alpha} \le$, $\beta < 0$.

According to the Schwarz inequality, $\mathrm{tr}(\widetilde{\Omega}^T(\Omega - \widetilde{\Omega})) \le \left\|\widetilde{\Omega}\right\|_F \|\Omega\|_F - \left\|\widetilde{\Omega}\right\|_F^2$.

Since $k_{\min}\|\xi\|^2 \le \xi^TK\xi$, $k_{\min}$ is the minimum eigenvalue of K, (38) can be expressed as:

$$
\begin{aligned}
\dot{V} \;\le\; & -k_{\min}\|\xi\|^2 + \varpi_N\|\xi\| \\
& +n\,\|\xi\|\,(\left\|\widetilde{\Omega}\right\|_F \|\Omega\|_F - \left\|\widetilde{\Omega}\right\|_F^2) + \beta \\
\le\; & -\|\xi\|\,(k_{\min}\|\xi\| - \varpi_N + n\left\|\widetilde{\Omega}\right\|_F(\left\|\widetilde{\Omega}\right\|_F - \Omega_M)) + \beta
\end{aligned} \tag{40}
$$

If $k_{\min}\|\xi\| - \varpi_N + n\left\|\widetilde{\Omega}\right\|_F(\left\|\widetilde{\Omega}\right\|_F - \Omega_M) = k_{\min}\|\xi\| - \varpi_N + n(\left\|\widetilde{\Omega}\right\|_F - \frac{1}{2}\Omega_M)^2 - \frac{n}{4}\Omega_M^2 \ge 0$, then $\dot{V} \le 0$.

Therefore, $\|\xi\| \ge \frac{\varpi_N + \frac{n}{4}\Omega_M^2}{k_{\min}}$, or $\left\|\widetilde{\Omega}\right\|_F \ge \frac{1}{2}\Omega_M + \sqrt{\frac{\Omega_M^2}{4} + \frac{\varpi_N}{n}}$ must be satisfied.

From the convergence of $\|\xi\|$, we find that proper adjustment of n and $k_{\min}$ improves the tracking accuracy.

4. Simulation Results

Some simulations for the flexible-joint manipulator (1) are conducted in this section to verify the effectiveness of the proposed controller by selecting $k_1 = 1.75$, $k_2 = 1.75$, $k_3 = 1.75$, $k_4 = 1.75$, $n = 0.1$, $\eta = 150$, $\Gamma_1 = 250$, $\Gamma_2 = 250$, $\Gamma_3 = 250$, the order of Chebyshev polynomials is 3. Other parameters are consistent with those in [16]. The initial state $x = [\,0 \;\; 0 \;\; 0 \;\; 0\,]^T$. The desired trajectory is set as $q_{\mathrm{ld}} = 0.2\sin(t)$.

Figures 1–4 show the simulation results. The responses of the system states q_l, $\dot{q}_l$, q_r, $\dot{q}_r$ are displayed in Figs. 1 and 2. The proposed controller achieves satisfactory control results and all the variables in the closed-loop system are bounded from Figs.1 and 2.

To demonstrate the superior performance of our proposed control method, we conducted a comparative analysis with a popular existing approach [21], which utilises RBF NNs for approximating unknown functions. Fig. 3 displays the trajectories [q_l (CNN) and q_l (RBF)] of the proposed and wildly used method (RBF), and the desired signal q_{ld}. From Fig. 4, we can find the excellent performance of the proposed controller. To quantitatively characterise performance, we calculated the maximum tracking errors at steady state separately when two different controllers were used, and the maximum tracking errors at steady state are, respectively, 0.0043 and 0.010.

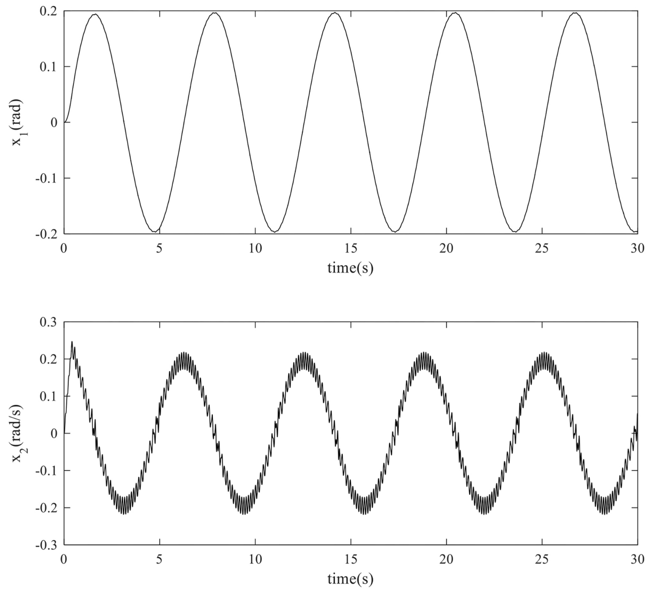

Figure 1. Responses of q_l and $\dot{q}_l$.

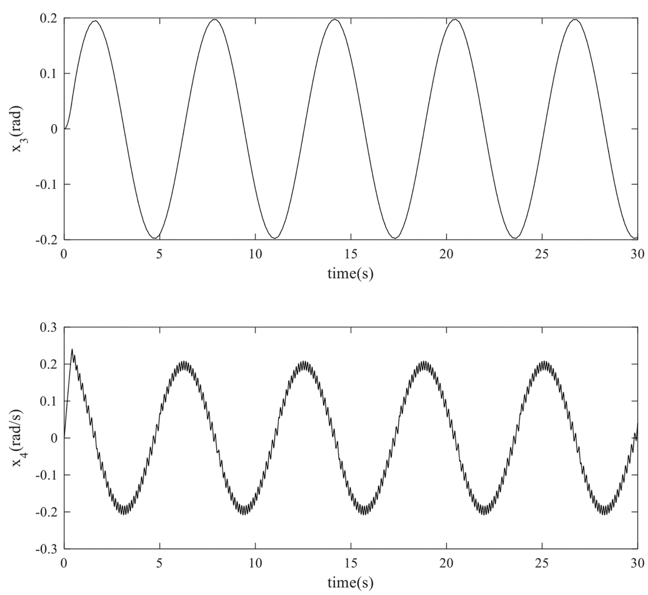

Figure 2. Responses of q_r and $\dot{q}_r$.

It can be seen that the controller proposed in this paper is easier to find the right parameters to make the control more accurate. We also calculated ITAE values separately for both controllers. The ITAE value of the proposed controller is 1.2748, and the ITAE value of the RBF controller is 1.8028, which validate the superior performance achieved by utilising the proposed controller.

5. Conclusion

In this paper, a novel trajectory tracking controller is proposed for flexible-joint manipulator, which is subject to time-varying and mismatched uncertainties, by integrating backstepping and CNN. The CNN are used to approximate the unknown functions. The adaptive law for CNN is derived from Lyapunov stability analysis. The proposed

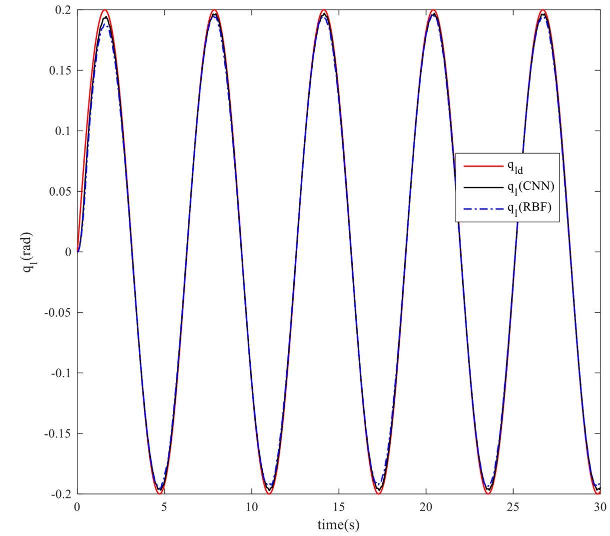

Figure 3. Trajectory tracking performance of the two controllers.

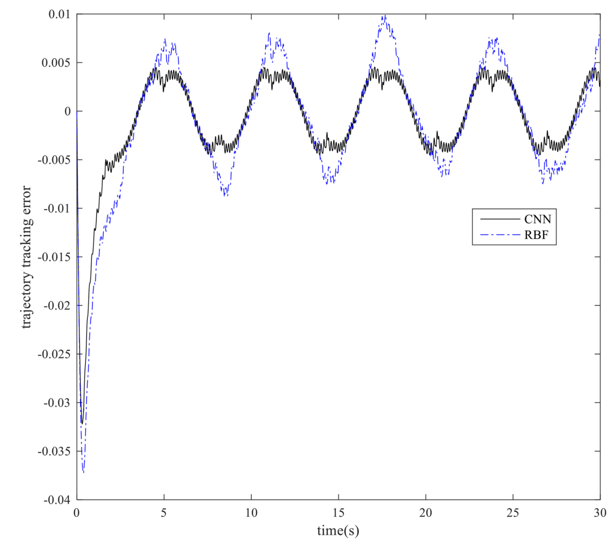

Figure 4. Trajectory tracking error for two controllers.

controller ensures that all the signals in the closed-loop systems are bounded. The comparative simulation experiments validate the superior performance of the proposed method over the commonly used RBF NN. However, it is challenging to use the same strategy directly for the tracking control of an n-link flexible-joint manipulator with mismatched uncertainties. Therefore, the next step in our research is to design trajectory tracking controller for n-link flexible-joint manipulator with mismatched uncertainties.

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