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OPTIMAL POINT TARIFFS IN TRANSMISSION AND DISTRIBUTION USING NONNEGATIVE LEAST SQUARES ESTMATES

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ABSTRACT
Point tariffs with node, zone or system specific injection and extraction charges are fairly common in power systems, for instance in Scandinavia, where in addition, price equalization is also rather extensive even across a large part of the transmission and distribution system. The paper examines theoretical transmission charges and marginal costs. Their relation to optimal transmission pricing, as determined by full optimal load flow (OPF) models, is examined and leads to an overdetermined set of linear equations. The pricing method proposed and used to determine injection and extraction prices is a nonlinear least squares model. A simple power system representation is examined in the form of a numerical example and implications are discussed for larger, real life size systems in terms of how the overall transmission cost is affected, based on the ideal marginal cost as represented by the nodal Lagrange multipliers of OPF.

KEY WORDS
Transmission, pricing, tariffs, optimization, cost

1. Introduction

Point tariffs for electricity transmission pricing generally utilize node specific injection and extraction (in-feed/out-feed) charges. However, in many instances the charges are levelled out for whole zones or regions to obtain equalized and uniform transmission cost to generators and consumers across the system. This equalization may be for political or other purposes but it may also be preferred to use point tariffs rather than nodal prices for other reasons. Bilateral contracts generally link generators with distribution companies, loads or other entities which join in such exchanges and contracts within the system. In particular, the Scandinavian countries, including Iceland, seem to use these form of contracts, seemingly often with heavy cross subsidies in order to obtain uniformity in charges. This even may result in one injection and one extraction price across the system.

In this paper we develop a framework to define point tariffs to minimize the deviation from system wide nodal prices which can be derived form optimal power flow (OPF). Bilateral contracts in the form of power injections and loads (extractions) at the system buses are thereby assumed. The concept of *point tariff* is, for the purpose of this paper, defined as an *array of nonnegative injection and extraction charges* to and from each network node, perhaps with a given specified uniformity within zones or regions. These node specific nonnegative in/out charges are a key point, since they cannot accurately represent "negative" or positive transmission costs in the form of congestion rent across the system. The problem at hand is to let these charges represent the costs as accurately and closely as possible. Therefore we want to determine the "optimal" charges, based on minimizing some measure of deviation from an "ideal" pricing scheme based on optimal power flow with nodal prices. Therefore, on the one hand, an absolute value of accumulated deviations can be chosen to minimize the measure of deviations. On the other hand a quadratic measure or a *least squares* measure is defined based on the overall deviation in total welfare involving all contracts. In this paper we define – as a part of an ongoing project – the appropriate quantities, present the key concepts, develop a model which is tested in a simple numerical example and finally discuss the implication of the results for further research or development.

The paper is organized as follows. In section 2 the optimization least squares model is presented involving notation and computational procedures. In Section 3 the modelling framework is illustrated with simple demonstrative numerical examples. In section 4 we discuss the applicability of the method and results and the needs for further research and extension of the present framework. Finally there is a section with references.

2. The Modelling Framework

Assume a power system with N nodes, indexed $i \in \{1, 2, ...N\}$, In general we assume the possibility of partitioning the system into K zones, indexed $k \in \{1, 2, ...K\}$, where $K \leq M$ and where we assume a *uniform* point tariff for all nodes belonging to the same zone. However, to simplify the present discussion, assume node specific charges in the following. Further assume any bilateral contract with injection at node *i* and extraction at node *j* involving quantity u_{ij} ([1]). The total injected power into node *i* is therefore $U_i = \sum_{j=1}^{N} u_{ij}$ and the total extracted power from node *j* is $V_j = \sum_{i=1}^{N} u_{ij}$. Further assume a given OPF model based on system wide assumptions specified for the purpose of establishing representative costs for point tariff charges. Such analysis will result in system wide nodal *Lagrange multipliers* which represent nodal prices λ_i for each node. Therefore the marginal cost difference between any injection node *i* and extraction at node *j or* $\lambda_{ij} = \lambda_i - \lambda_j$ can under ideal market conditions be interpreted as the marginal transmission cost (congestion rent). This in turn is the "optimal" transmission price scheme under ideal conditions as present in the previously mentioned OPF model. Since for the marginal transmission cost $\lambda_{ij} = -\lambda_{ji}$ we have a skew symmetric square matrix of transmission prices/marginal costs, or:

$$\boldsymbol{\lambda} = \begin{bmatrix} 0 & -\lambda_{12} & \cdots & -\lambda_{1N} \\ \lambda_{12} & 0 & & -\lambda_{2N} \\ \vdots & & \ddots & \vdots \\ \lambda_{1N} & \lambda_{2N} & \cdots & 0 \end{bmatrix} \quad (1)$$

Therefore "ideal" transmission costs may be negative under the given operating condition, reflecting the possibility of counter flows in contracts set against the main power flow where the counter flow reduces cost leading to a negative marginal price.

Under ideal market conditions, the total transmission charges for all bilateral contracts, using the marginal cost matrix (1) as a basis for pricing, would be C_{TXMCOST} where:

$$C_{\text{TXMCOST}} = \sum_{i=1}^{N} \sum_{j=1}^{N} u_{ij} \lambda_{ij} \quad (2)$$

Note that a given set of bilateral contracts u_{ij} may be added on top of any basic power flow (OPF) where the resulting OPF leads to (1). From (1) and (2) we want to establish a point tariff, where each node has an injection and an extraction charge. In addition we generally require that these unit charges per MW or MWh are nonnegative, which means that the parties involved in the bilateral contract, both generators and consumers, should pay to the grid operator, TSO, rather than receive reimbursement.

Assume now that a point tariff is applied to the bilateral contracts where a unit injection charge at node *i* is r_i and s_j is the unit extraction charge at node *j*. Then we get the following expression for the total charges according to the point tariff.

$$C_{\text{PT}} = \sum_{i=1}^{N} \sum_{j=1}^{N} u_{ij} (r_i + s_j) \quad (3)$$

It can easily be seen that the point tariff cannot generally reflect the above matrix, (1) especially for a system of realistic size, when the price equalization in specified for a given zone. For the charges in the point tariff to reflect accurately the costs in (1) we must have the following condition. It means that the sum of injection charges at node *i* and extraction charges at node *j* for any given bilateral contract should equal the ideal transmission cost, or:

$$r_i + s_j = \lambda_{ij} \qquad \forall i, j \quad (4)$$

Here λ_{ij} are given constants from OPF models and r_i and s_j are variables. This condition, (4) of course, leads to $C_{\text{TXMCOST}} = C_{\text{PT}}$ and certainly cannot be fulfilled for the diagonal elements of (1) unless the charges are either zero or negative.

We can choose to ignore the diagonal line in (1). By doing so (4) becomes a system of $N \times (N-1)$ linear equations with 2*N* variables and it only has a solution if $2N \le N^2 - N$ which means that an accurate point tariff can be implemented only for 2 or 3 buses. We may also choose to impose a tariff on a contract where the injection bus is the same as the extraction bus. Then we have 2*N* variables and N^2 equations meaning that the diagonal of (1) is active.

Even for a relatively small number of buses, (4) represents an *over-determined* system of linear equations and by imposing additional *nonnegative constraints*, (4) does not have a feasible solution due to the skew symmetric nature of the marginal cost matrix (1).

Therefore our problem involves an overdetermined set of linear equations with nonnegative constraints:

$$\begin{aligned} u_{ij}(r_i + s_j) &= u_{ij}\lambda_{ij} \qquad \forall i, j \\ r_i \ge 0 \;;\; s_i &\ge 0 \qquad \forall i, j \end{aligned} \quad (5)$$

This set, (5) can be rewritten:

$$\mathbf{Ax} = \mathbf{b} \qquad \mathbf{x} \ge \mathbf{0} \quad (6)$$

where the "solution" vector, $\mathbf{x}$, represents the estimated injection and extraction charges, the matrix, $\mathbf{A}$ is composed of 1's and 0's multiplied by weighting factors u_{ij} introduced as the size of the bilateral contract with an

[1] For instance real average power in MW during a specific period or kWh.

injection at bus i and extraction at bus j. Finally, the vector, $\mathbf{b}$ represents the marginal nodal prices times the size (energy amount) of the contracts, or the total monetary transaction.

Since no analytical and exact solution exists with the constraints of the nonnegative charges, the problem is a *nonnegative least squares problem* defined as follows:

Minimize $\|\mathbf{Ax}-\mathbf{b}\|$ subject to $\mathbf{x} \geq \mathbf{0}$
where:
$\mathbf{A}$ is a $m \times n$ known coefficient matrix, $m \geq n$,
$\mathbf{b}$ is the m element vector with known constants, and
$\mathbf{x}$ is the n element unknown solution vector. (7)

In (7) $m = N^2$ or $m = N^2 - N$ is the number of equations, depending on whether injecting and extracting at the same bus is charged for, and $n = 2N$ is the number of variables. N is the number of nodes as previously defined.

The norm $\|\mathbf{d}\|$ may be either the sum of squares of deviations or the sum of absolute values of the deviations, $\mathbf{d} = \mathbf{Ax} - \mathbf{b}$. Other norms are possible. For each bilateral contract, the deviations from the ideal transmission charges are $d_{ij} = u_{ij}(r_i + s_i - \lambda_{ij})$. Using "sum of the squares", (least squares) the optimal point tariffs will be determined by minimizing the following objective function:

$$C_{\mathrm{DEV1}} = \sum_{i=1}^{N}\sum_{j=1}^{N}\left\{d_{ij}\right\}^2 = \sum_{i=1}^{N}\sum_{j=1}^{N}\left\{u_{ij}(r_i + s_i - \lambda_{ij})\right\}^2 \quad (8)$$

Subject to the constraints:

$$r_i \geq 0 \;\text{ and }\; s_i \geq 0 \;\; \forall i \quad (9)$$

However, with the deviations as absolute values, we get:

$$C_{\mathrm{DEV2}} = \sum_{i=1}^{N}\sum_{j=1}^{N}\left|d_{ij}\right| \quad (10)$$

Subject to:

$$d_{ij} = u_{ij}(r_i + s_i - \lambda_{ij}) \quad (11)$$

$$r_i \geq 0 \;\text{ and }\; s_i \geq 0 \;\; \forall i \quad (12)$$

The above formulation (10) - (12) can be represented as a standard Linear Programming (LP) problem by modifying the objective as follows:

$$C_{\mathrm{DEV2}} = \sum_{i=1}^{N}\sum_{j=1}^{N}\left(d_{ij+} + d_{ij-}\right) \quad (13)$$

Subject to the following constraints:

$$d_{ij+} + d_{ij-} = u_{ij}(r_i + s_i - \lambda_{ij}) \quad (14)$$

$$r_i \geq 0 \;\text{ and }\; s_i \geq 0 \;\; \forall i \quad (15)$$

$$d_{ij+} \geq 0 \;\text{ and }\; d_{ij-} \geq 0 \;\; \forall i \quad (16)$$

In (8)-(16) λ_{ij} and u_{ij} are of course specified constants, while all other quantities are unknown variables.

3. Numerical Cases and Examples

We exemplify the above discussion by presenting 2 basic simple systems and numerical cases for these systems. The nonnegative least squares approach is first applied to a simple 2 bus system.

3.1 A simple 2 bus system.

Assume the almost a trivial system in Figure 1. It is desired to determine 2 charges for each node, injection and extraction or a total of 4 variables.

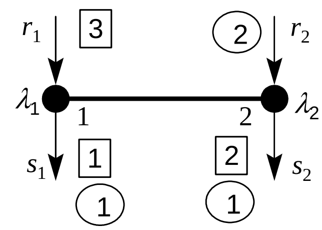

Figure 1: A simple 2-node system

Assume the marginal transmission cost is given from a simple optimal power flow (OPF) as a set of Lagrange multipliers for the 2 nodes and we have in this case: $\lambda_{12} = \lambda_1 - \lambda_2 = 2$ units and therefore $\lambda_{21} = -2$ units in this example. Further assume bilateral contracts with an injection of 3 units at bus 1 with 2 units extracted at bus 2 and 1 unit extracted at bus 1. Similarly, an injection of 2 is assumed at bus 2 where half is transmitted to bus 1 and half is extracted at bus 2. Therefore, $u_{12} = 2$, $u_{11} = 1$ and $U_1 = 3$ etc. and the squares show on set of contracts and the circles another set.

We write out the linear equations of (5) and (6) below:

$$\begin{aligned} r_1 \;+\; s_1 &= 0 \\ 2r_1 \;+2s_2 &= 4 \\ r_2 \;+\; s_1 &= -2 \\ r_2 \;+\; s_2 &= 0 \end{aligned} \quad (17)$$

$$r_1 \geq 0\;, s_1 \geq 0\;, r_2 \geq 0\;, s_2 \geq 0 \quad (18)$$

resulting in (with this order of variables)

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 2 & 0 & 0 & 2 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} 0 \\ 4 \\ -2 \\ 0 \end{bmatrix} \quad (19)$$

It is easy to see that the system has a singular matrix and the following Matlab[2] commands:

```
A=[1 1 0 0;2 0 0 2;0 1 1 0;0 0 1 1]
b=[0;4;-2;0]
d=det(A)
x = lsqnonneg(A,b)
```

result in:

```
d =
```

[2] ©Mathworks Inc.

```
x =      0
    0.8889
         0
         0
    0.8889
```

The Matlab function **`lsqnonneg`** is used to calculate the least squares non-negative solution:

$$\boxed{r_1 = 0.8889 \;\; , s_1 = 0 \;\; , \; r_2 = 0 \;\; , s_2 = 0.8889} \qquad (20)$$

For any solution, since we have a singular matrix, we have an infinite number of positive or negative solutions and in fact any solution $r_1 = z \;\; , s_1 = -z$, $r_2 = z - 2 \;\; , s_2 = 2 - z$, where z is an arbitrary positive or negative real number.

For instance, we can choose to solve this directly by minimizing the sum of the squares and get the following expression:

$$\begin{aligned} C_{\text{DEV1}} &= [2(r_1 + s_2 - 2)]^2 + [1(r_1 + s_1 - 0)]^2 \\ &+ [1(r_2 + s_1 + 2)]^2 + [1(r_2 + s_2 - 0)]^2 \end{aligned} \qquad (21)$$

To minimize C_{DEV1} , we simplify and differentiate C_{DEV1} and solve by Newton's method, i.e. set the derivative to zero and thereby get the following system of linear equations:

$$\begin{array}{rrrrl} 10r_1 & +2s_1 & & +\;8s_2 & = 16 \\ 2r_1 & +4s_1 & +2r_2 & & = -4 \\ & 2s_1 & +4r_2 & +\;2s_2 & = -4 \\ 8r_1 & & +2r_2 & +10s_2 & = 16 \end{array} \qquad (22)$$

This system (22) has the form $\mathbf{Ax} = \mathbf{b}$ where

$$\mathbf{A} = \begin{bmatrix} 10 & 2 & 0 & 8 \\ 2 & 4 & 2 & 0 \\ 0 & 2 & 4 & 2 \\ 8 & 0 & 2 & 10 \end{bmatrix} \qquad (23)$$

and $\mathbf{b}^T = [16 \quad -4 \quad -4 \quad 16]$ and it leads to the solution : $\mathbf{x}^T = [-0.8999 \quad 0.8999 \quad -2.8999 \quad 2.8999]$ (using Matlab). Therefore we have the following solution: $r_1 = -0.9$, $s_1 = 0.9$, $r_2 = -2.9$, $s_2 = 2.9$. As we see the direct unconstrained solution leads to negative injection charges and the injection charges are -0.9 and -2.9 units respectively at the 2 buses while the extraction charges are similarly 0.9 and 2.9 units respectively. These solutions are an instant to the variable z as indicated above.

3.2 A simple 3 bus system.

Now we consider a numerical example in a 3 bus system as shown in the Figure, (shown without injections, etc):

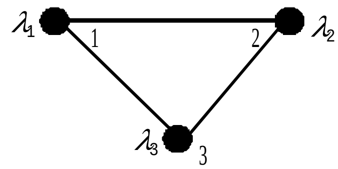

Figure 2: A simple 3-node system

Assume that $\lambda_1 = 1$; $\lambda_2 = 2$; $\lambda_3 = 3$ and all contracts are for simplicity sake equal or $u_{ij} = 1$ unit from one node i to node j . Then we have the following set of equations:

$$\begin{array}{llllllll} r_1 & +s_1 & & & & & = 0 \\ r_1 & & & + s_2 & & & = -1 \\ r_1 & & & & & +s_3 & = -2 \\ & & r_2 & +s_2 & & & = 0 \\ & s_1 & +r_2 & & & & = 1 \\ & & r_2 & & & +s_3 & = -1 \\ & s_1 & & & +r_3 & & = 2 \\ & & & s_2 & +r_3 & & = 1 \\ & & & & r_3 & +s_3 & = 0 \end{array} \qquad (24)$$

The following Matlab commands:

```
A=[1 1 0 0 0 0;1 0 0 1 0 0;1 0 0 0 0 1;
    0 0 1 1 0 0;0 1 1 0 0 0;0 0 1 0 0 1;
    0 1 0 0 1 0;0 0 0 1 1 0;0 0 0 0 1 1]
b=[0;-1;-2;0;1;-1;2; 1; 0]
x= lsqnonneg(A,b)
```

will result in the following output (rearranged for simplicity and compactness):

```
A =
     1     1     0     0     0     0
     1     0     0     1     0     0
     1     0     0     0     0     1
     0     0     1     1     0     0
     0     1     1     0     0     0
     0     0     1     0     0     1
     0     1     0     0     1     0
     0     0     0     1     1     0
     0     0     0     0     1     1
```

```
b =
     0
    -1
    -2
     0
     1
    -1
     2
     1
     0
```

```
x =
         0
    0.7500
         0
         0
    0.7500
         0
```

This means that: $\boxed{r_3 = 0.75 \text{ and } s_1 = 0.75 \text{ and other charges are zero}}$. If we for instance relax the requirement of the diagonal line in λ_{ij} by setting a price for the input/output at the same bus in the first, 4th and the last equation of (24), for instance 0.2, 0.4 and 0.6 units respectively, we get (rearranged for simplicity and compactness):

b =	x =
0.2000	0
-1.0000	0.7500
-2.0000	0
0.4000	0
1.0000	0.9500
-1.0000	0
2.0000	
1.0000	
0.6000	

This means that: $r_3 = 0.75$ while $s_1 = 0.95$ and other charges are zero so it does not alter basically the structure of the least squares solution. This concludes our discussion of the simple numerical examples.

4. Conclusion and Discussion

Finally in this section we discuss the model and its practical applicability and results and suggest further research and testing of the method. How does the above discussion fit into tariff design for transmission and distribution systems? The following points are an attempt to summarize this:

- **Optimal instantaneous marginal costs**. With the node specific in/out restriction imposed on the point tariff structure, we have analyzed how it is possible to approximate the optimal real time nodal prices using least squares with in/out charges. We have summarized the suggested methodology and tested it to some extent on very simple examples.
- **Snapshot of operating conditions**. We have only tested a snapshot based on a hypothetical OPF with given instantaneous operating conditions leading to the associated OPF and nodal prices. What is needed is an expanded temporal framework where the nodal prices are transformed to point tariff charges based on this methodology [2]. This should be one of our topics for an extended research and testing.
- **Economic efficiency**. An important topic is the ability of the tariff arrangement to help overall economic efficiency in the short and long run. The current methodology does not consider an overall economic efficiency , such as computing the charges to be paid to the TSO and how they will support the transmission system fixed cost.
- **Fixed cost recovery**. It is therefore necessary to be able to recover the fixed investment costs for the system, which may or may not be recovered considering only the marginal costs. The above discussion therefore should be expanded to consider the relation of marginal to average costs in a transmission system and how point tariffs fit into that picture. This should be another topic for an extended research and testing.
- **Wider range spatial equalization of charges**. Finally, as previously mentioned, to get an accurate picture of the longer range transmission costs, it is necessary to consider the dynamics on a minute to days or weeks time scale and obtain perhaps a longer range equalization of charges. Similarly, in the introductory discussion the possibility of defining zones with equalized prices remains to be investigated. Hopefully this is one more topic feasible for further research on this method of determining point tariff charges.

Therefore, to summarize the objective and individual steps for further investigation and testing of the method presented here, we present a summary and an outline of the methodology below. This is a topic to be investigated in further research as a continuation on this paper's main topic.

a) Define a system of realistic size with tens, hundreds or thousands of buses and define an appropriate time period with time steps.
b) Define the basic operating conditions such as a basic load and basic bilateral contracts on top of that basic load as injections and extractions of specified MWh's in each time step (on top of that basic load).
c) Run an OPF for all time steps and obtain Lagrange multipliers for each node and each time step as nodal marginal prices.
d) Consider other requirements such as recovering fixed cost for each transmission link or for the system as a whole and how this condition is merged with the problem definition and model formulation.
e) Run the least squares calculation considering the zonal partitioning in the system, if any, and adaptation to covering the fixed costs recovery during the time frame selected.
f) Interpret the results by calculating the total charges and how these total charges compare for instance with marginal prices and total transmission costs accumulated for all transaction.

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