

## COMPLETELY DECENTRALIZED STATE ESTIMATION FOR ACTIVE DISTRIBUTION NETWORK

P. H. Nguyen, W. L. Kling, J. M. A. Myrzik  
Department of Electrical Engineering, Eindhoven University of Technology  
5600MB Eindhoven  
The Netherlands  
[p.nguyen.hong@tue.nl](mailto:p.nguyen.hong@tue.nl); [w.l.kling@tue.nl](mailto:w.l.kling@tue.nl); [j.m.a.myrzik@tue.nl](mailto:j.m.a.myrzik@tue.nl)

### ABSTRACT

The concept of an active network has been mentioned recently to adapt to the large-scale implementation of distributed generation. Distributed State Estimation is one of the most important functions to enable this novel concept. This paper proposes a completely decentralized state estimation (CDSE) method suitable for the active network. CDSE is based on multi-agent system technology to compute iteratively state variables, i.e., voltage magnitude and angle. The method will be compared with the classical weighted least square method in term of computation burden. As examples a 3-bus system and the IEEE 14-bus network will be investigated.

### KEY WORDS

Distribution system, state estimation, multi-agent system, active network, distributed generation.

### 1. Introduction

Encouragement to develop more distributed generation (DG) challenges the distribution networks in coping with bidirectional power flows, voltage variations, fault level increases, protection selectivity, power quality and stability. The concept of an active network (AN) has been mentioned recently for the distribution system to adapt to the large-scale implementation of DG [1]. With one more control layer, each local area network is defined as a cell that can manage power inside and across cell's boundaries. Therefore, the power flow or bus voltages are controlled in an efficient, flexible and intelligent way that can overcome the problems of the existing distribution system.

Under the control architecture of the AN, distributed state estimation (DSE) plays a vital role to enable those control actuators. Depending on control stages, i.e., locally (voltage regulation) or globally (power flow control), the DSE processes its own real-time and pseudo measurement information, and coordinates with other neighbour DSEs to get whole network state variables. Note that pseudo measurements have to be used occasionally due to lacking real-time measurement in the distribution networks.

State estimation (SE) was firstly introduced with the classical weighted least square (WLS) method [2]. In an effort to reduce computation burden, several hierarchical state estimation methods were proposed [3]. Under the power system deregulation with emerging tasks for the network operator, the DSE has become a critical research [4].

In [5], the authors introduced a robust distributed algorithm for overlapping bus boundaries. It is implemented with an introduction of linearized augmented Lagrangians. The method gave one of the most practical and realistic performances in distributed state estimation. In [6], a straightforward but effective method for overlapping tie-line boundaries was presented. After estimating local state variables by the WLS, boundary state variables ( $V_i^b$  and  $\theta_i^b$ ) are exchanged among the sub-areas and each area solves its local problem again. The algorithm stops if the boundary state variables do not change significantly.

The DSE based on multi-agent system (MAS), i.e., an application of information and communication technologies, was presented in [7]. By exchanging messages among substation agents, the method has shown significant advantages in state estimation computing and bad data detection steps. However, the research has just been concerned on novel aspects, i.e., illustrating a feasibility of the concept with current sensors.

In a different approach, an ultra fast state estimation for the large electric power system was presented in [8]. Given at each bus a microprocessor, the bus state variables can be calculated by processing local bus information and its neighbours. With supporting of a strong communication system, the method can increase computing speed significantly.

This paper proposes a completely decentralized state estimation (CDSE) method that is elaborated from two last above methods. CDSE is based on MAS technology to compute iteratively the state variables, i.e., voltage magnitude and angle. The effectiveness and computation burden of the method will be compared with the classical

WLS. As examples a 3-bus system and the IEEE 14-bus network will be investigated.

## 2. Methodology

### 2.1 Background of State Estimation

As the most common method used for SE, the classical WLS aims to find the log-likelihood function by solving following problem [8]:

$$\text{Minimize } J(x) = [z - h(x)]^T R^{-1} [z - h(x)] \quad (1)$$

where:

- $x$  system state vector,
- $z$  measurement vector,
- $h$  function vector relating measurements to system state,
- $R$  variance vector of the measurement errors.

The application of Gauss-Newton method for non-linear optimal condition leads to an iterative solution as shown below:

$$\Delta x^{k+1} = [H^T(x^k) \cdot R^{-1} \cdot H(x^k)]^{-1} \cdot H^T(x^k) \cdot R^{-1} \cdot [z - h(x^k)]$$

where:

$$H(x^k) = \frac{\partial h(x^k)}{\partial x^k} \quad \text{is the Jacobean matrix.}$$

Computing a gain matrix  $H^T(x^k) \cdot R^{-1} \cdot H(x^k)$  for the large-scale power system is extremely heavy, which mitigates the application of the WLS method. Hence, several improvements, i.e., decoupled formulation or DC estimation, were proposed to reduce the computation burden.

For the same purpose, the DSE represents above centralized state estimation problem (1) by decentralized state estimation problems as follows [6]:

$$\text{Minimize } \sum_{a=1}^n J(x_a) + \sum_{a=1}^n \sum_{b \in B(a)} J(x_a, x_b) \quad (2)$$

where:

- $x_a$  local state vector,
- $x_b$  boundary state vector.

In general, the DSE includes two steps, i.e., local state estimation and coordination. Requirement of measurement data for interconnection lines between the sub-networks is a necessary condition. According to the way of defining the sub-networks (including tie-lines or not), the DSE has particular solutions regarding information exchange and objective of the coordination layer [5-6].

Although including two state estimation steps, the core algorithm of (2) is still based on the function of  $J(\cdot)$ , that will be solved by the WLS method. These algorithms can be implemented in parallel but still depend on the particular local area network. In case of having large and

complicated sub-networks, the number of iteration steps is significant.

### 2.2 Completely decentralized state estimation

In this section, a different approach, the so-called completely decentralized state estimation method (CDSE), is proposed. The basic idea of the CDSE method was firstly mentioned in [8]. Regarding each line  $i$ - $j$ , the non-linear power flow equations are presented as follows:

$$P_{ij} + jQ_{ij} = f_{ij}(V_i, V_j, \theta_i, \theta_j) \quad (3)$$

From (3), the state variables  $(\theta_i, V_i)$  of bus  $i$  can be computed by measurement data from bus  $j$  with variances  $\tau$  and  $\sigma$ , respectively. These are denoted as the normal distribution in following equations:

$$V_i^j = N(V_i, \sigma_{ij}^2); \quad \theta_{ij} = N(\theta_j - \theta_i, \tau_{ij}^2) \quad (4)$$

where:

$$j = 1, \dots, M_i; i = 1, \dots, N.$$

Depending on requirements of accuracy and computation speed, sets of information in (4) contaminated random noise will be integrated in different grades to obtain a smoothed estimate.

While the grade 1 give over-simplified algebraic equations to estimate ultra fast state variables, the grade 3 considers all correlations of variables in a control center computer to get the most accurate values.

Regarding independence of measurement errors, the grade 2 computes variables based on maximum likelihood estimation as follows:

$$V_i^* = \frac{\frac{1}{\sigma_{ii}^2} V_i^i + \sum_{j=1}^{M_i} \frac{1}{\sigma_{ij}^2} V_i^j}{\frac{1}{\sigma_{ii}^2} + \sum_{j=1}^{M_i} \frac{1}{\sigma_{ij}^2}} \quad (5)$$

$$\theta_{ij}^* = \frac{\frac{1}{\tau_{ij}^2} \theta_{ij} + \frac{1}{\tau_{ji}^2} \theta_{ji}}{\frac{1}{\tau_{ij}^2} + \frac{1}{\tau_{ji}^2}} \quad (6)$$

Equation (5) and (6) represent the least square estimate of the voltage at bus  $i$  and angle differences from bus  $i$  to its neighbor buses. The grade 2 estimation can be obtained very fast with an acceptable accuracy. In addition, it can be implemented straightforward in a distributed way.

The CDSE method uses the grade 2 estimation as a basic formulation. Iteration steps are used at each bus to compare estimated values with previous step values. This technique is similar with the iteration technique in [6]. However, state variables are updated locally after each iteration. Consequently, CDSE can get a more accurate solution in a completely distributed way. CDSE is deployed by supporting of MAS. The deployment will be mentioned in Section 3.

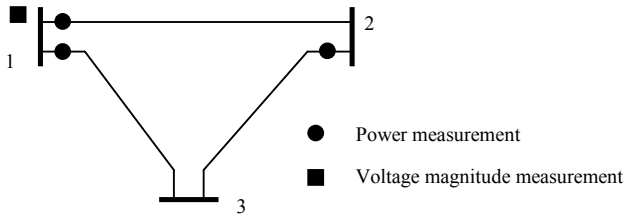


Figure 1. Single diagram of 3-bus system.

The proposed method is explained by investigating a simple case of a 3-bus system, shown in Figure 1. The network data and measurement data are modified from [9] and summarized in Table 1 and 2, respectively.

In the example, each bus  $i$  starts initializing its local state variables ( $V_i; \theta_i$ ) to typical values, i.e., values from previous state, flat values, or available measurement data. The state vector and associated variance vector are:

$$x^T = [\theta_1, \theta_2, \theta_3, V_1, V_2, V_3] = [0, 0, 0, 1.006, 1, 1]$$

$$\varepsilon^T = [\tau_1, \tau_2, \tau_3, \sigma_1, \sigma_2, \sigma_3] = [0.1e-3, 1, 1, 4e-3, 1, 1]$$

At bus 1, the state variables are estimated by its own information as follows:

$$V_1^1 = N(V_1, \sigma_{11}^2) = N(1.006, 1.6e-5)$$

Regarding line 1-2, the state variables of bus 1 are estimated using information on line power measurements ( $P_{12}, Q_{12}$ ) and pseudo-measurements of bus 2 ( $V_2, \theta_2$ ). Note that these pseudo-measurements have significant variance compared with real measurement. As the size of this 2-bus 1-branch network is very small, the WLS method can be applied to estimate ( $V_1^2, \theta_{12}$ ). In addition, the diagonal elements of the gain matrix  $H^T(x^k).R^{-1}.H(x^k)$  provides variance ( $\sigma_{12}, \tau_{12}$ ) for further calculation [12].

Table 3 and 4 summarize the state variables estimated by different buses and the least square estimate values. After the first iteration, the new state vector and associated variance vector are:

$$x^{T(1)} = [0, -0.0213, -0.0475, 1.006, 0.9805, 0.9507]$$

$$\varepsilon^{T(1)} = [0.1, 0.44, 0.86, 5.7, 5.8, 6.0] \cdot 10e-3$$

The algorithm stops when the new state variables are close to the previous values. Table 5 summarizes the final results of the proposed algorithm and compares with conventional WLS method.

### 3. MAS-based implementation

One of main reasons mitigating the application of proposed method in [8] is a lacking of strong computing technology. In the same manner, the CDSE method needs a suitable technology supporting distributed and parallel computation for each bus. This paper introduces MAS as a possible technology to overcome these challenges.

The MAS technology is based on the concept of intelligent agents, which are defined as entities (software or hardware) able to react to changes in their

environments and to interact with other agents [10]. A possible application for MAS to manage the AN is illustrated in Figure 2 [11].

Table 1. Physical data of 3-bus test network.

From	To	R	X	B
1	2	0.01	0.03	0
1	3	0.02	0.05	0
2	3	0.03	0.08	0

Table 2. Measurement data of 3-bus test network.

Measurement $i$ ,	Type	Value, pu	$\sqrt{R_{ii}}$ , pu
1	P12	0.888	0.008
2	P13	1.173	0.008
3	P23	0.386	0.008
4	Q12	0.568	0.008
5	Q13	0.663	0.008
6	Q23	0.228	0.008
7	V1	1.006	0.004

Table 3. Estimated voltages.

Bus	$V_i^1; \sigma_{ij}$		
	1	2	3
1	1.006; 6.3e-2	1.025; 0.9e2	1.058; 1.0e2
2	0.981; 6.4e-2	1.000; 1.0e2	1.029; 1.0e2
3	0.951; 6.5e-2	0.975; 1.0e2	1.000; 1.0e2

Table 4. Estimated bus angles.

Line	$\theta_{ij}; \tau_{ij}$	$\theta_{ji}; \tau_{ji}$
1-2	-0.021; 1.8e-2	0.024; 1.8e1
1-3	-0.048; 2.4e-2	0.041; 3.0e2
2-3	-0.025; 2.2e2	0.023; 2.1e2

Table 5. State estimation results for 3-bus test network.

Bus	WLS		CDSE (3 iterations)	
	V [p.u.]	Phase [rad]	V [p.u.]	Phase [rad]
1	1.006	0	1.006	0
2	0.9805	-0.0213	0.9806	-0.0212
3	0.9507	-0.0474	0.9507	-0.0475

The application of CDSE method can be integrated in the control structure of the AN, shown in Figure 3. Each agent (moderator)  $A_i$  can get its own information through measurements. The agent  $A_i$ , then, sends its information to the neighbor  $A_j$  through the MAS platform. After getting its own information and neighbor information, each agent  $A_i$  can calculate local state variables based on (5) and (6).

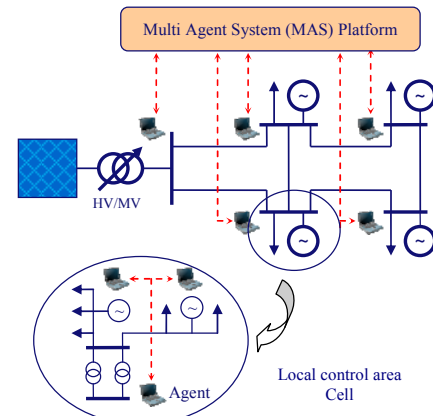


Figure 2. Active Network managed by multi-agent system.

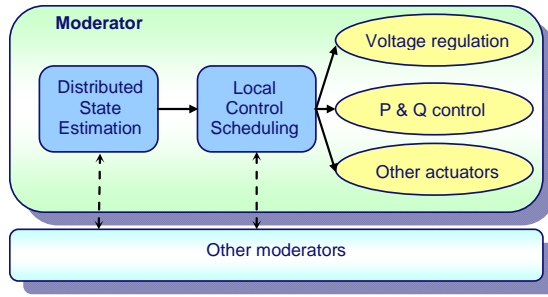


Figure 3. Control architecture of the moderator.

Behaviors of the agent are summarized in the Table 6.

Table 6. Pseudo-Code for  $A_i$  actions.

```

Mode ← Received_message(objective)
Switch Mode {
Case 1:
Initialize()
 $A_i$  ← Measurements ( $V_i, \theta_{ij}, P_{ij}, Q_{ij}$ )
 $A_i$  ← Update ( $V_i, \theta_{ij}, P_{ij}, Q_{ij}$ )
Send Inform( $A_i, A_j, V_i, \theta_{ij}, P_{ij}, Q_{ij}$ )
Case 2:
( $V_i^j; \sigma_{ij}^2$ ), ( $\theta_{ij}; \tau_{ij}^2$ ) ← Receive Inform( $A_j, A_i, V_j, \theta_{ij}, P_{ij}, Q_{ij}$ )
If ( $j \geq M_i$ ) then
( $V_i^*; \theta_{ij}^*$ ) ← ( $V_i^j; \sigma_{ij}^2; \theta_{ij}; \tau_{ij}^2$ );  $\forall j \in M_i$ 
If ( $V_i^*; \theta_{ij}^*$ ) = ( $V_i^-; \theta_{ij}^-$ );  $\forall j \in M_i$  then STOP
Else Send Inform( $A_i, A_j, V_i^*, \theta_{ij}^*, P_{ij}, Q_{ij}$ )
Else Wait( $A_{j+i}$ )
End

```

## 4. Case studies

The proposed method is implemented with Matlab/Simulink. The IEEE 14-bus network [13], shown in Figure 4, is investigated with several case studies. The network data is summarized in Table 7. The effectiveness and computation speed of the algorithm are compared with the conventional WLS method.

### 4.1 Redundant measurements

The proposed method was investigated with redundant measurements, i.e., 4 voltage bus measurements (at bus 1, 8, 9, and 10) and power measurements on all branches (20 branches). Measurement data are shown in Table 8 and 9.

Results of the proposed method were compared with the conventional WLS method and true values in Table 10. After 7 iterations, the state variables can be obtained by the proposed method. Estimated values are quite close to the true values, which are calculated from normal power flow calculation program.

Table 7. Physical data of IEEE 14-bus test network.

From	To	R	X	B
1	2	0.0194	0.0592	0.0528
1	5	0.0540	0.2230	0.0492
2	3	0.0470	0.1980	0.0438

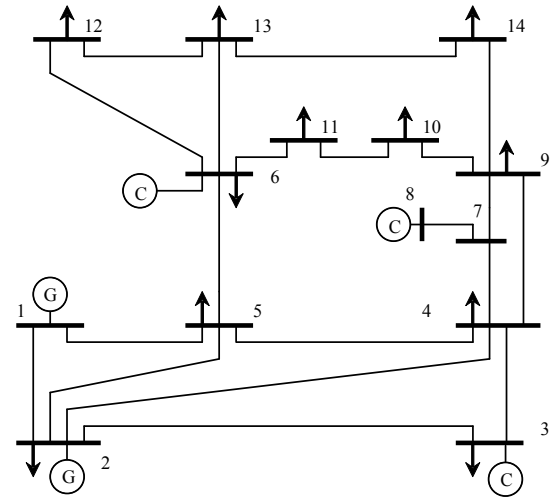


Figure 4. Single diagram of IEEE 14-bus system.

2	4	0.0581	0.1763	0.0340
2	5	0.0570	0.1739	0.0346
3	4	0.0670	0.1710	0.0128
4	5	0.0134	0.0421	0.0000
4	7	0.0000	0.2091	0.0000
4	9	0.0000	0.5562	0.0000
5	6	0.0000	0.2520	0.0000
6	11	0.0950	0.1989	0.0000
6	12	0.1229	0.2558	0.0000
6	13	0.0662	0.1303	0.0000
7	8	0.0000	0.1762	0.0000
7	9	0.0000	0.1100	0.0000
9	10	0.0318	0.0845	0.0000
9	14	0.1271	0.2704	0.0000
10	11	0.0821	0.1921	0.0000
12	13	0.2209	0.1999	0.0000
13	14	0.1709	0.3480	0.0000

Table 8. Power measurement data.

From	To	Active power flow		Reactive power flow	
		Value, pu	$\sqrt{R_{ii}}$ , pu	Value, pu	$\sqrt{R_{ii}}$ , pu
1	2	1.5591	0.008	-0.2018	0.008
1	5	0.7648	0.008	-0.0372	0.008
2	4	0.5602	0.008	-0.0912	0.008
3	2	-0.7020	0.008	0.0133	0.008
4	7	<b>0.2842</b>	<b>0.008</b>	<b>-0.1195</b>	<b>0.008</b>
4	9	<b>0.1610</b>	<b>0.008</b>	<b>-0.0296</b>	<b>0.008</b>
5	6	<b>0.4374</b>	<b>0.008</b>	<b>-0.1192</b>	<b>0.008</b>
6	11	0.0710	0.008	0.0498	0.008
6	12	0.0779	0.008	0.0269	0.008
6	13	0.1764	0.008	0.0795	0.008
8	7	0.0000	0.008	0.2084	0.008
9	10	0.0549	0.008	0.0282	0.008
9	14	0.0953	0.008	0.0271	0.008
14	13	-0.0548	0.008	-0.0253	0.008
11	10	0.0354	0.008	0.0305	0.008
2	5	0.4147	0.008	-0.0832	0.008
3	4	-0.2400	0.008	-0.0404	0.008
5	4	0.6288	0.008	-0.0705	0.008
7	9	0.2842	0.008	0.0638	0.008
12	13	0.0162	0.008	0.0094	0.008

Table 9. Voltage measurement data.

Measurement $i$ ,	Type	Value, pu	$\sqrt{R_{ii}}$ , pu
1	V1	1.06	0.004
2	V8	1.09	0.004
3	V9	1.051	0.004
4	V10	1.0462	0.004

**Table 10.** State estimation results with redundant measurements.

Bus	True values		WLS		CDSE (7 iterations)	
	$V$ [p.u.]	$\theta$ [rad]	$V$ [p.u.]	$\theta$ [rad]	$V$ [p.u.]	$\theta$ [rad]
1	1.060	0.000	1.059	0.000	1.059	0.000
2	1.045	-0.086	1.046	-0.087	1.045	-0.087
3	1.010	-0.220	1.008	-0.220	1.007	-0.220
4	1.031	-0.182	1.033	-0.183	1.032	-0.184
5	1.036	-0.156	1.038	-0.158	1.037	-0.158
6	1.070	-0.256	1.072	-0.257	1.070	-0.257
7	1.056	-0.237	1.057	-0.237	1.057	-0.238
8	1.090	-0.237	1.090	-0.237	1.090	-0.238
9	1.050	-0.265	1.051	-0.266	1.050	-0.266
10	1.046	-0.268	1.047	-0.269	1.046	-0.270
11	1.055	-0.264	1.055	-0.265	1.055	-0.266
12	1.055	-0.271	1.056	-0.271	1.055	-0.272
13	1.050	-0.272	1.051	-0.273	1.050	-0.273
14	1.032	-0.286	1.033	-0.286	1.032	-0.287

**Table 11.** State estimation results with limited measurements.

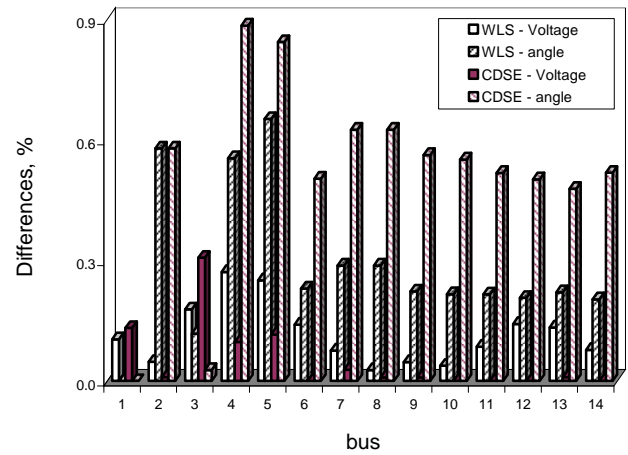
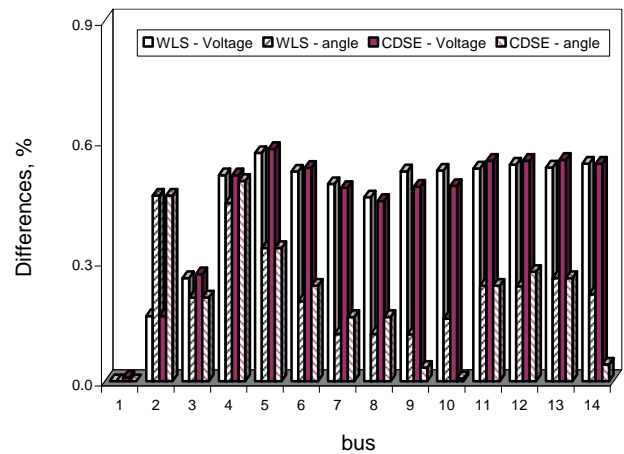
Bus	True values		WLS		CDSE (4 iterations)	
	$V$ [p.u.]	$\theta$ [rad]	$V$ [p.u.]	$\theta$ [rad]	$V$ [p.u.]	$\theta$ [rad]
1	1.060	0.000	1.060	0.000	1.060	0.000
2	1.045	-0.086	1.047	-0.087	1.047	-0.087
3	1.010	-0.220	1.007	-0.220	1.007	-0.220
4	1.031	-0.182	1.036	-0.183	1.036	-0.183
5	1.036	-0.156	1.042	-0.157	1.042	-0.157
6	1.070	-0.256	1.076	-0.256	1.076	-0.256
7	1.056	-0.237	1.062	-0.237	1.061	-0.237
8	1.090	-0.237	1.095	-0.237	1.095	-0.237
9	1.050	-0.265	1.056	-0.265	1.055	-0.265
10	1.046	-0.268	1.052	-0.268	1.051	-0.268
11	1.055	-0.264	1.060	-0.264	1.060	-0.264
12	1.055	-0.271	1.060	-0.270	1.061	-0.270
13	1.050	-0.272	1.055	-0.271	1.055	-0.271
14	1.032	-0.286	1.037	-0.285	1.037	-0.285

The percentage differences of the estimated values from the true values are depicted Figure 5. Comparing with the conventional WLS method, the proposed method gives more accurate voltage magnitude values while giving less accurate bus angle values.

#### 4.2 Limited measurements

The proposed method was investigated with limited measurements, i.e., 1 voltage bus measurements (at bus 1) and power measurements on 14 branches. Measurement data are shown in Table 8 and 9 without italic font style.

Results of the proposed method were compared with the conventional WLS method and true values in Table 10. In this case, the method takes only 4 iterations to estimate the state variables. Due to lacking of voltage measurements, the differences of the estimated voltage magnitude increase. However, the differences of bus angles decrease. It can be explained by the reduced number of unnecessary measurements that might cause more noise in the estimating procedure. These differences of voltage magnitude and bus angles are depicted in Figure 6.

**Figure 5.** Differences of state variables from true values – Redundant measurement case.**Figure 6.** Differences of state variables from true values – limit measurement case.

#### 4.3 Lacking critical measurements

WLS requires the whole network to be observable, which can be determined by a spanning tree. Lacking critical measurements makes the network unobservable or partial observable. In this case, the WLS method can not solve the problem.

The CDSE method has the advantage to obtain a solution for partial observable networks. This advantage is investigated with lacking critical measurement data of the IEEE 14-bus system, i.e., no measurement between the high voltage and the low voltage network. Measurement data are shown in Table 8 and 9 without bold font style.

It can be seen from Table 12, that CDSE can estimate bus angles with acceptable errors. As providing sufficient voltage measurement data, the error of estimated voltage magnitude is quite small. The method takes only 6 iterations in this case.

**Table 12.** State estimation results with unobservable measurements.

Bus	True values		CDSE (6 iterations)		Error	
	V	$\theta$	V	$\theta$	$\Delta V$	$\Delta\theta$
	[p.u.]	[rad]	[p.u.]	[rad]	[p.u.]	[rad]
1	1.060	0.000	1.060	0.000	0.000	0.000
2	1.045	-0.086	1.047	-0.087	-0.002	0.000
3	1.010	-0.220	1.010	-0.219	0.000	-0.001
4	1.031	-0.182	1.036	-0.183	-0.005	0.001
5	1.036	-0.156	1.041	-0.157	-0.005	0.001
6	1.070	-0.256	1.070	-0.157	0.000	-0.099
7	1.056	-0.237	1.056	-0.183	0.000	-0.054
8	1.090	-0.237	1.090	-0.183	0.000	-0.054
9	1.050	-0.265	1.050	-0.183	0.000	-0.082
10	1.046	-0.268	1.046	-0.186	0.000	-0.082
11	1.055	-0.264	1.055	-0.166	0.000	-0.099
12	1.055	-0.271	1.055	-0.172	0.000	-0.099
13	1.050	-0.272	1.050	-0.173	0.000	-0.099
14	1.032	-0.286	1.032	-0.204	0.000	-0.082

#### 4.4 Convergence analysis

Basically, the proposed method is also using WLS technique to estimate local voltage and angle differences. However, scale of computation matrix with only two interactive buses and interconnection line is much smaller than central SE and DSE. In addition, each typical bus of the power system are connected normally with maximum four other buses. Therefore, the processor of each bus can get convergence within few loops. Distributed and parallel working of processor improves computation time significantly.

This algorithm has an advantage when its iteration steps have not been influenced by the scale of whole network.

#### 5. Conclusion

This paper proposes a completely decentralized state estimation (CDSE) method for the Active Network. The proposed method can obtain an accurate solution comparable with conventional WLS method. It can be straightforward implemented in a distributed way with supporting of MAS. In addition, the method can give an acceptable solution in case of unobservable network. It can be seen from these results, that CDSE is able to work locally within cells or globally for the whole AN.

Further study will concern more about bad data detection of the proposed method. The dynamic simulation that can show more effect of MAS will be also implemented.

#### References

[1] F.V. Overbeeke, Active networks: Distribution networks facilitating integration of distributed generation, *Proc. 2<sup>nd</sup> international symposium on distributed generation: power system and market aspects*, Stockholm, 2002.

[2] F.C. Schweppe, & J. Wildes, Power System Static-State Estimation, Part I: Exact Model, *Transaction on Power Apparatus and Systems*, 89(1), 1970, 120-125.

[3] Th.V. Cutsem, & M. Ribbens-Pavella, Critical survey of Hierarchical Methods for State Estimation of Electric Power Systems, *IEEE Transactions on Power Apparatus and Systems*, PAS-102(10), 1983, 3415-3424.

[4] M. Shadidehpour, & Y. Wang, *Communication and Control in Electric Power Systems – Applications of parallel and distributed processing* (IEEE Press, NJ: John Wiley & Sons, Inc., 2003).

[5] R. Ebrahimiyan, & R. Baldick, State Estimation Distributed Processing, *IEEE Transactions on Power System*, 15(4), 2000, 1240-1246.

[6] A.J. Conejo, S.d.L. Torre, & M. Canas, An Optimization Approach to Multiarea State Estimation, *IEEE Transactions on Power System*, 22(1), 2007, 213-221.

[7] M.M. Nordman, & M. Lehtonen, Distributed Agent-Based State Estimation for Electrical Distribution Networks, *IEEE Transactions on Power Systems*, 20(2), 2005, 652-658.

[8] J. Zaborszky, K.W. Whang, & K.V. Prasad, Ultra Fast State Estimation for the Large Electric Power System, *IEEE Transactions on Automatic Control*, AC-25(2), 1980, 839-841.

[9] A. Abur, & A.G. Exposito, *Power System State Estimation: Theory and Implementation*, (Marcel Dekker, Inc., 2004).

[10] S.D.J. McArthur, E.M. Davidson, V.M. Catterson, A.L. Dimeas, N.D. Hatzigiorgiou, F. Ponci, & T. Funabashi, Multi-Agent Systems for Power Engineering Applications - Part I: Concepts, Approaches, and Technical Challenges, *IEEE Transactions on Power Systems*, 22(4), 2007, 1743-1752.

[11] P.H. Nguyen, W.L. Kling, & J.M.A. Myrzik, Coordination of Voltage Regulation in Active Networks, *Proc. IEEE Transmission and Distribution Conference and Exposition*, Chicago, 2008.

[12] K. Li, State Estimation for Power Distribution System and Measurement Impacts, *IEEE Transactions on Power System*, 11(2), 1996, 911-916.

[13] University of Washington, Power System Test Case Archive, Available Online: <http://www.ee.washington.edu/research/pstca/>.