A FEEDBACK LINEARIZATION BASED GENERATOR VOLTAGE REGULATOR FOR POWER SYSTEMS

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ABSTRACT

A new method of designing a nonlinear voltage regulator is proposed in this paper. The regulator is based on output feedback linearization and can achieve both voltage regulation and stability enhancement simultaneously. Conventionally output feedback linearization has been used for voltage regulator design by taking infinite bus voltage as reference. This paper proposes to derive the control law using the output feedback linearization approach by taking the secondary voltage of the step-up transformer as reference instead of the infinite bus voltage. It has been possible to considerably improve the damping performance of the proposed regulator with this modification. The performance of the proposed regulator is evaluated on a single machine infinite bus model of a power system at diverse operating conditions. The simulation results show that the performance of the proposed regulator is better or comparable to that of a linear static AVR equipped with a power system stabilizer and always better than that of a conventional nonlinear voltage regulator. A linearized analysis has been included for a better understanding of the proposed regulator. As the regulator design is based on local measurements with no reference to external system parameters it could potentially be used in a multi-machine environment.

KEY WORDS

Power System Stability, Input-Output feedback linearization, small-signal stability, Transient Stability.

1 Introduction

Generator excitation controllers play an important role in the stabilization of power systems. They are primarily designed to regulate the terminal voltage during normal and post fault operating conditions. Modern fast acting, high gain automatic voltage regulator (AVR) also enhances the overall transient stability of the system. However, a power system stabilizer (PSS) as an auxiliary controller is often needed for damping the low frequency oscillations [1, 2]. Traditionally the AVR and PSS controllers have been designed based on linearized models of the power system. The linearized models on which the controllers are based, depend upon the system operating condition. Any significant deviation from this nominal operating condition can considerably degrade the performance of the controllers.

The fixed gain stabilizers perform reasonably well if they have been tuned properly [3]. Though these stabilizers have simple robust structures, tuning them is an involved process which requires considerable expertise and also a knowledge of system parameters external to the generating station [2, 4]. These parameters may not be readily available and may vary during normal operation of the power system. In order to overcome these difficulties several researchers have considered nonlinear control techniques such as feedback linearization (FBL), variable structure control etc. [5–10] for excitation system design. These controllers aim to replace the existing AVR+PSS structure, but the benefits of this replacement remains to be properly established.

A nonlinear controller which can achieve voltage regulation and system stability simultaneously by using inputoutput feedback linearization has been proposed in [11]. In [11] the terminal voltage of the machine is taken as an output to derive the control law for a Single Machine Infinite Bus system (SMIB). Under nominal operating conditions the transient stability and voltage profile is greatly improved with this control law but small signal stability performance is not as satisfactory as that of a static linear AVR+PSS controller. It is observed that under weak system conditions the system response becomes more oscillatory even for small disturbances. Though it is mentioned in [11] that the tuning of a gain parameter can improve the performance, it is found that this gain can be varied only over a narrow range.

This paper proposes a nonlinear voltage regulator design by taking the secondary voltage of the step-up transformer (High-Voltage Bus) as reference instead of the infinite bus voltage. This permits an assessment of system disturbances such as changes in system configuration or load variations based on the deviations in power flow and voltage at the high voltage bus of the step-up transformer [12]. It is observed that the performance of the proposed nonlinear AVR is usually better or comparable to that of a static linear AVR+PSS over a wide range of operating and system conditions. The proposed control law has been linearized to understand this behavior of the controller under various operating conditions. It is found that the structure of the linearized controller is similar to that of a static linear AVR with an additional component due to deviations in rotor speed. It is this component that causes additional rotor damping. This component of damping, however decreases with increasing transmission line impedance and loading in a conventional nonlinear AVR. In the proposed controller the additional damping component remains practically constant in spite of changes in system and operating conditions which accounts for its better damping performance.

2 Power System Modeling

Appropriate dynamic modeling of major power system components such as the synchronous generator, excitation system, AC network etc. are required for stability analysis. In this paper IEEE Model 1.0 has been used to represent the synchronous generator [13]. This results in a classical third order dynamic model for a SMIB power system. The use of this model is justified and a large number of nonlinear excitation controllers are designed based on this model [14]. The system dynamic equations are given by

$$\dot{\delta} = w_B S_m \tag{1}$$

$$\dot{S}_m = \frac{1}{2H} \{ T_{mech} - T_{elec} - DS_m \}$$
 (2)

$$\dot{E}'_{q} = \frac{1}{T'_{do}} \left\{ -E'_{q} + (X_{d} - X'_{d})i_{d} + E_{fd} \right\}$$
(3)

where

$$T_{elec} = E'_q i_q + (X'_d - X_q) i_d i_q \tag{4}$$

and terminal voltage V_t is given by,

$$V_t = \sqrt{V_d^2 + V_q^2} \tag{5}$$

The direct and quadrature axis voltages V_d and V_q can be expressed as,

$$V_d = E'_d - X_q i_q - R_a i_d \tag{6}$$

$$V_q = E'_q + X'_d i_d - R_a i_q \tag{7}$$

The variables have standard meaning [13]. Here the control variable is the field voltage E_{fd} . The objective is to design a nonlinear control law for E_{fd} to get proper voltage regulation i.e. terminal voltage $V_t \rightarrow V_{t0}$ a pre specified voltage.

3 Design of Nonlinear Voltage Regulator

Output feedback linearization technique essentially results in a controller that makes part of the system dynamics to behave in a linear manner [11]. The linear dynamics to be achieved in SMIB is the terminal voltage of the generator. The control law derived in [11] is briefly explained in this section. The state variables for SMIB are defined



Figure 1. A Single Machine connected to external network

as $x = \{\delta, S_m, E'_q\}$. Derivatives of the output V_t are taken successively until the control input E_{fd} appears in the equation. The total number of derivatives taken is called the relative degree of the system and in this case relative degree is one. Defining the tracking error as $e = V_t - V_{t0}$, a fixed gain parameter $K_v > 0$ can be chosen such that the error dynamics $\dot{e} + K_v e = 0$ as $t \to \infty$. By solving the error dynamics equation one can arrive at the following nonlinear control law for E_{FD} . The details of the derivation can be found in [11].

$$E_{fd} = \frac{T'_{do}}{C_3 V_q} \begin{bmatrix} -K_v \left(V_t - V_{t0} \right) \sqrt{V_t} - \left(V_d C_{11} \cos \delta w_B \right) \\ S_m E_b \right) + V_q C_{22} E_b \sin \delta w_B S_m \end{bmatrix} + E'_q - \left(X_d - X'_d \right) i_d$$
(8)

where

$$C_{11} = \frac{X_q}{X_e + X_q}, C_{22} = \frac{X'_d}{X_e + X'_d}, C_{33} = \frac{X_e}{X_e + X'_d}$$
(9)

It is to be noted that K_v is the only gain parameter that is to be tuned for improvement in the performance. This control law gives the necessary closed-loop control which, when applied, causes the output, terminal voltage, of the machine to behave linearly [11].

4 Proposed Approach

In this paper a single machine connected to an external power system as shown in Fig.1 is considered for the nonlinear AVR design. The rotor angle with respect to the voltage $V_s \angle \theta_s$ of the high voltage bus is defined as $\delta_s = \delta - \theta_s$ and is given by

$$\delta_s = tan^{-1} \quad \frac{P_s \left(X_t + X_q \right) - Q_s \left(R_a + R_t \right)}{P_s \left(R_a + R_t \right) + Q_s \left(X_t + X_q \right) + V_s^2} \tag{10}$$

where $P_s = V_s I_a \cos \theta_p$, $Q_s = V_s I_a \sin \theta_p$ and θ_p is power factor angle at the high voltage bus. In rare cases, under leading power factor operations

$$P_s(R_a + R_t) + Q_s(X_t + X_q) + V_s^2 < 0$$
 (11)

and δ_s is given by

$$\delta_s = \pi - |\delta_s \text{ obtained in (10)}| \tag{12}$$

The expressions for E'_q , i_d , i_q , V_d and V_q in terms of δ_s are as follows [12, 15]

$$E'_{q} = \frac{\left(X_{t} + X'_{d}\right)}{X_{t}} \sqrt{V_{t}^{2} - \left(\frac{X_{q}}{\left(X_{t} + X_{q}\right)}V_{s}\sin\delta_{s}\right)^{2}} - \frac{X'_{d}}{X_{t}}V_{s}\cos\delta_{s}$$
(13)

The machine terminal voltage in terms of the transformer secondary is given by

$$V_Q + jV_D = (V_q + jV_d)e^{j\delta}$$

= $(i_q + ji_d)(R_t + jX_t)e^{j\delta} + V_s \angle \theta_s$ (14)

The subscripts q and d refers to the q and d-axis respectively in Park's reference frame and Q and D refers to the q and d-axis respectively in Kron's reference frame.

$$(V_q + jV_d) = (i_q + ji_d)(R_t + jX_t) + V_s \angle \theta_s \ e^{-j\delta}$$

Replacing δ by $\delta_s + \theta_s$ and equating the real and imaginary parts of the above equation gives the modified stator algebraic equations referred to the transformer bus as shown below. These equations are true even in multi machine environment for each machine.

$$V_q = R_t i_q - X_t i_d + V_s \cos \delta_s$$

$$V_d = R_t i_d + X_t i_q - V_s \sin \delta_s$$
(15)

Using (6), (7) in (15) and simplifying one can get the following

$$i_{d} = \frac{V_{s} \cos \delta_{s} - E'_{q}}{X'_{d} + X_{t}}; \quad i_{q} = \frac{V_{s} \sin \delta_{s}}{X_{q} + X_{t}}$$
$$V_{d} = C_{33s}E'_{q} + C_{22s}V_{s} \cos \delta_{s}; \quad V_{q} = -C_{11}V_{s} \sin \delta_{s}$$
(16)

where

$$C_{11s} = \frac{X_q}{X_t + X_q}, C_{22s} = \frac{X'_d}{X_t + X'_d}, C_{33s} = \frac{X_t}{X_t + X'_d}$$

Now the expression for terminal voltage is given by

$$V_t = \sqrt{\left(C_{33s}E'_q + C_{22s}V_s\cos\delta_s\right)^2 + \left(C_{11s}V_s\sin\delta_s\right)^2}$$
(17)

Exactly following the procedure explained in Section-3, using the assumption $\dot{\delta_s} = w_B S_m$ and neglecting the variation of V_s one can arrive at the following modified nonlinear control law.

$$E_{fd} = \begin{cases} -K_v \left(V_t - V_{t0} \right) \sqrt{V_t} \\ - \begin{bmatrix} V_d C_{11s} \cos \delta_s \\ + V_q C_{22s} \sin \delta_s \end{bmatrix} V_{s0} w_B S_m \end{cases} \frac{T'_{do}}{C_{33s} V_q} \\ + E'_q - \left(X_d - X'_d \right) i_d \tag{18}$$

The subscript '0' indicates initial operating condition. The constants C_{ii} 's (i = 1 to 3) are independent of external reactance X_e . The proposed controller can assess the system disturbances such as changes in system configuration or load variations based on the deviations in δ_s computed from the power flow and voltage at the high voltage bus of the step-up transformer. This control law can be easily realized by computing (10), (13) and (16) from P_s , Q_s and V_s measurements at the high voltage bus.

5 Simulation Results

The performances of the conventional nonlinear AVR (8) and the proposed AVR (18) have been extensively evaluated on a well known SMIB system analyzed in [1]. The system performance is also tested using a static linear AVR, with and without a power system stabilizer. The system and stabilizer parameters are given in [15]. Results of only a few representative test cases are shown in Fig.2 to Fig.4. In these figures CFBL and PFBL refer to the conventional and the proposed feedback linearized controller.



Figure 2. δ response, 10% Step change in V_{ref}

Fig.2 shows the responses of the nominal SMIB system $(P_t + jQ_t = 1 + j0.2p.u., V_t = 1p.u., X_e = 0.4p.u.)$ in terms of the rotor angle δ for a 10% step change in V_{ref} . The system is operated with (a) Conventional nonlinear AVR, $K_v = 20$, (b) Linear AVR + PSS, (c) Proposed nonlinear AVR, $K_v = 20$ and d) Linear AVR without PSS. The system is unstable with a linear AVR alone. The system becomes stable with both conventional and proposed nonlinear AVRs and their performances are comparable to that of the linear AVR+PSS controller.

Fig.3 shows the responses of the SMIB with same conditions as above, following a 3ϕ fault cleared after 50ms by tripping one of the parallel lines. After the fault is cleared the system becomes weak with an equivalent external reactance $X_e = 0.7$. The system is more oscillatory with conventional nonlinear AVR. The performance of the



Figure 3. δ response, 3ϕ fault at transformer bus, cleared by line tripping

proposed AVR is slightly better than the performance of the static AVR+PSS.



Figure 4. δ response, 10% step change in T_m , Weak system

Fig.4 shows the system responses for weak system $(P_t + jQ_t = 1 + j0.5p.u., X_e = 0.8p.u, V_t = 1p.u.)$. The responses shown are for a 10% step change in T_m . In this case also the response of the system is unstable with the conventional nonlinear AVR. The proposed controller stabilizes the system and its performance is comparable to that of the linear AVR+PSS. The possible reasons for this remarkable performance of the proposed nonlinear AVR is investigated in the following sections.

6 Analysis of the Proposed Nonlinear AVR

In order to understand the behavior of the proposed controller at different operating conditions, control law (18) is linearized using the conventional Taylor series approximation. Linearization of (18) gives

$$\Delta E_{fd} = \frac{T'_{do}}{C_{33s}V_{q0}} \begin{bmatrix} -K_v \left(\Delta V_t - \Delta V_{ref}\right) \sqrt{V_{t0}} + \\ \left(V_{q0}C_{22s}V_{s0}\sin\delta_{s0}w_B \\ -V_{d0}C_{11s}V_{s0}\cos\delta_{s0}w_B\right) \Delta S_m \end{bmatrix} + \Delta E'_q - \left(X_d - X'_d\right) \Delta i_d \tag{19}$$

Linearizing V_t given in (5) one can get

$$\Delta V_t = K_5 \Delta \delta + K_6 \Delta E'_q \tag{20}$$

Similarly linearizing i_d given in (16) results in

$$\Delta i_d = C_1 \Delta \delta_s + C_2 \Delta E'_q + C_3 \Delta V_s \tag{21}$$

where

$$K_5 = -\frac{X_q V_{d0} E_b \cos \delta_0}{(X_q + X_e) V_{t0}} - \frac{X'_d V_{q0} E_b \sin \delta_0}{(X_e + X'_d) V_{t0}}$$
(22)

$$K_6 = \frac{X_e}{X_e + X'_d} \frac{V_{q0}}{V_{t0}};$$
(23)

$$C_1 = -C_{22s} V_{s0} \sin \delta_{s0}; \ C_2 = \frac{-1}{X_t + X'_d}$$
(24)

$$C_3 = C_{22s} \cos \delta_{s0}$$

substituting (20), (21) in (19) and rearranging the terms one can arrive at the equation (25) where $\Gamma_1 = V_{q0}V_{s0}w_BC_{22s}\sin\delta_{s0}$ and $\Gamma_2 = V_{d0}V_{s0}w_BC_{11s}\cos\delta_{s0}$. This equation can be rewritten as

$$\Delta E_{fd} = K_{FBL} \begin{bmatrix} -G_5 \Delta \delta_s - G_6 \Delta E'_q + \Delta V_{ref} \\ +G_D \Delta S_m - G_{V_s} \Delta V_s \end{bmatrix}$$
(26)

Fig.5 shows the block diagram representation of (26) and the block diagram of a static linear AVR is shown in Fig.6. From these block diagrams, the proposed controller can be interpreted as a high gain (K_{FBL}) , fast exciter with negligible delay. It has three components negatively affecting the torque angle loop, one due to the deviations in rotor angle $\Delta \delta_s$ denoted by G_5 , one due to the deviations in flux linkages $\Delta E'_q$ denoted by G_6 and one due to the deviations in voltage magnitude of the high voltage bus ΔV_s . It also contains an additional component G_D due to the deviations in slip speed S_m as shown with dashed circle in Fig.5. G_D contributes positively to the torque angle loop just like a power system stabilizer. The equivalent of G_5, G_6 for static linear AVR are denoted by K_5, K_6 which are standard Heffron-Phillip's parameters of a SMIB system. The block diagram of the conventional nonlinear controller is similar but with $V_s = E_b$, $G_{V_s} = 0$ and $\Delta \delta_s = \Delta \delta$.

Variations of parameters G_5 and G_D are plotted by varying generator power P_t from 0.5p.u. to 1.1p.u. for various values of X_e . The terminal voltage V_t is fixed at 1p.u. Fig.7 shows the variations of G_5 , K_5 with P_t . Gain K_v of the proposed controller is fixed at 20 so that the variation of G_5 and G_6 is almost same as that of the static linear

$$\Delta E_{fd} = \frac{T'_{do}K_v\sqrt{V_{t0}}}{C_{33s}V_{q0}} \begin{cases} -\underbrace{\left[K_5 + \frac{(X_d - X'_d)V_{q0}C_{33s}C_1}{T'_{do}K_v\sqrt{V_{t0}}}\right]}_{G_5} \Delta \delta_s - \underbrace{\left[-\frac{C_{33s}V_{q0}}{T'_{do}K_v\sqrt{V_{t0}}} + \frac{(X_d - X'_d)V_{q0}C_{33s}C_2}{T'_{do}K_v\sqrt{V_{t0}}} + K_6\right]}_{G_6} \Delta E'_q \\ +\Delta V_{ref} + \underbrace{\frac{(\Gamma_2 - \Gamma_1)}{K_v\sqrt{V_{t0}}}}_{G_D} \Delta S_m - \underbrace{\frac{(X_d - X'_d)V_{q0}C_{33s}C_3}{T'_{do}K_v\sqrt{V_{t0}}}}_{G_{V_s}} \Delta V_s \end{cases}$$

$$(25)$$



Figure 5. Output Feedback Linearization based AVR



Figure 6. Static linear AVR

AVR parameters K_5 and K_6 . For all foreseeable operating conditions, K_6 and G_6 are positive. K_5 and G_5 can be however positive or negative (see Fig.7). This has significant impact on system stability. The negative damping contribution of this arm has to be compensated by a PSS in a linear controller and by the G_D arm in the nonlinear controllers. For all operating conditions, G_{V_s} is small and therefore the effect of variation in voltage magnitude of the high voltage bus on system dynamic performance is not



Figure 7. Variation of G_5 , K_5 with P_g for various values of X_e



Figure 8. Variation of G_D with X_e of proposed nonlinear AVR and conventional nonlinear AVR

significant. Neglecting variations in V_s while deriving the proposed control law is thus justified.

Fig.8 shows the variation of G_D with P_t for different values of X_e . Solid lines show the variations for the conventional nonlinear AVR. Dashed lines show the variations for the proposed AVR. It can be seen that the damping component of the conventional FBL AVR reduces significantly with increase in external line impedance X_e and with increase in system loading. This is the reason for the conventional nonlinear controller's poor performance under weak system conditions and high loads.

If $G_D = 0$, the block diagram Fig.5 reduces to a very fast static linear voltage regulator shown in Fig.6 with $K_A = K_{FBL}$ and $T_A = 0$. Under this condition, the damping torque is positive whenever G_5 is positive but for a large number of cases G_5 is negative as shown in Fig.7. In the case of nonlinear regulator ($G_D \neq 0$), the G_D branch always contributes towards positive damping. The damping contribution of a nonlinear regulator is thus always more positive than that of a static linear voltage regulator. This positive contribution incase of the conventional nonlinear AVR reduces with increased loading and transmission line impedance as shown in Fig.8. Under these conditions G_5 is also more negative. The positive damping effect of the conventional nonlinear AVR is thus limited.

From the simulation results and above observations it can be inferred that the conventional nonlinear AVR may not be able to replace the linear AVR+PSS structure under high loads and weak system conditions because of the negative damping effect. The proposed approach for the nonlinear AVR design can overcome these difficulties and can replace the linear AVR+PSS structure as tuning a single parameter is always easier than tuning multiple parameters of a conventional power system stabilizers.

7 Conclusion

This paper has proposed and analyzed the performance of an output feedback linearization based nonlinear generator voltage regulator. It is found that the performance of conventional nonlinear AVR is always better than that of a static linear AVR comparable to that of a static linear AVR+PSS combination only over a limited range of operating conditions. The small signal stability performance is however, poor especially under weak system conditions. The proposed nonlinear voltage regulator controller overcomes these drawbacks. It has good transient stability performance and voltage regulation capability. The damping of small oscillations is also considerable improved. The implementation of this controller is very simple and it would be possible to use it in a multi-machine environment as it requires only local measurements and has a single tunable gain.

References

- F.P.Demello and C.Concordia, "Concepts of synchronous machine stability as affected by excitation control," *IEEE Trans. Power Apparatus and Systems*, vol. PAS-88, No.4, pp. 316–329, 1969.
- [2] E. Larsen and D. Swan, "Applying power system stabilizers, parts I,II and III," *IEEE Trans.Power Appara*-

tus and Systems, vol. PAS-100, pp. 3017–3046, June 1981.

- [3] G. Rogers, *Power System Oscillations*. (Kluwer Academic Publishers, 2000).
- [4] P. Kundur, M. Klein, G. Rogers, and M. Zywno, "Application of power system stabilizers for enhancement of overall system stability," *IEEE Trans.Power Syst.*, vol. 4,No.2, pp. 614–626, May 1989.
- [5] Q.Lu and Y.Sun, "Nonlinear stabilizing control of multimachine systems," *IEEE Trans. Power Syst.*, vol. 4, pp. 236–241, 1989.
- [6] J. W. Chapman, M. D. Ilic, C.A.King, L.Eng, and H.Kaufman, "Stabilizing a multimachine power system via decentralized feedback linearizing excitation control," *IEEE Trans. Power Syst.*, vol. 8, pp. 830– 839, 1993.
- [7] Y. Cao and O.P.Malik, "A nonlinear variable structure stabilizer for power system stability," *IEEE Trans. on Energy Conversion*, vol. 9, no.3, pp. 489–495, September 1994.
- [8] C.Zhu, R.Zhou, and Y. Wang, "A new nonlinear voltage controller for power systems," *Int. J. Electr. Power and Energy Syst.*, vol. 19, pp. 19–27, 1997.
- [9] J. David, Y. Guo, M. Larsson, and Y. Wang, *Global Control of Complex Power Systems*, ser. Lecture Notes in Control and Information Sciences. Springer Berlin / Heidelberg, 2004, vol. 293.
- [10] B. K. Kumar, S. Singh, and S. Srivastava, "A decentralized nonlinear feedback controller with prescribed degree of stability for damping power system oscillations," *Electric Power Systems Research*, vol. 77, Issues 3-4, pp. 204–211, March 2007.
- [11] F. K. Mak, "Design of nonlinear generator excitors using differential geometric control theories," in *Proc.* 31st IEEE Conf. Decision Control, Tuscon, AZ, 1992, pp. 1149–1153.
- [12] M. Nambu and Y. Ohsawa, "Development of an advanced power system stabilizer using a strict linearization approach," *IEEE.Trans.on Power Systems*, vol. 11,no.2, pp. 813–818, May 1996.
- [13] K.R.Padiyar, *Power System Dynamics Stability and Control.* (John Wiley; Interline Publishing, 1996).
- [14] Q. Lu, Y. Sun, and S. Mei, Nonlinear Control Systems and Power System Dynamics. Kluwer Academic, 2001.
- [15] G. Gurunath and I. Sen, "A modified heffron-phillips model for the design of power system stabilizers," in *IEEE PowerCon-2008*, New Delhi, India, October 12 - 15, 2008.