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A BACKWARD METHOD FOR SOLVING PV NODES IN WEAKLY MESHED DISTRIBUTION NETWORKS

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ABSTRACT
In this paper, a new iterative backward/forward methodology for the load flow solution of weakly meshed systems with fixed voltage nodes is presented. Such technique models the loads at each iteration by means of impedances, and the PV nodes by means of reactances.
Each iteration is organised in two steps. In the first, the radial network attained from the meshed system through cuts and composed of shunt and series impedances is solved. In the second step, based on a reduced Thévenin impedance matrix, the compensation currents to be injected in the cut nodes are deduced. Modelling the PV nodes by means of reactances allows the attainment, for the reactive power of these nodes, of the same precision that it is possible to get solving the network with the methods usually adopted for transmission networks. After the presentation of the different implementations of the backward/forward method proposed in the literature, the new technique and the relevant implementation are presented in detail. The methodology allows the solution of systems with meshes and PV nodes; test results show precision, speed and good convergence properties.

KEY WORDS
Backward/forward methods, distribution networks, and load flow.

1. Introduction

The most commonly used solution method for radial distribution networks is the iterative backward/forward (b/f) method ([1],[2], [3]). Its main points are: ease of implementation, robustness, good convergence properties, possibility to take into account any dependency of loads on voltage, limited computational burden. With reference to the current summation method, the b/f technique is articulated in two steps: the backward and the forward sweeps. In the backward sweep, based on the loads currents calculated starting from a given voltage profile, the branch currents can be evaluated starting from the terminal branches and going up to the root. In the forward sweep, starting from the source node with fixed voltage the voltages at all the nodes are calculated. If the loads are with constant current, the process stops with the calculation of the bus voltages; differently, based on the dependency of the loads currents on voltage, the new values of the currents are calculated and another backward phase is carried out. In other variants of the method, in the backward phase the branches power flows or the driving points admittances, seen from the nodes downstream, are evaluated. The forward phase repeats the sequential calculation of the bus voltages. The iterative process stops when a prefixed convergence condition is verified.
If the network is meshed, the solution process proceeds through the ‘radialization’ of the networks by means of a number of cuts that equals the number of independent meshes ([4], [5], [6]). At the couple of nodes created by the cut, the injection of two equal and opposite currents is imposed, and its value is determined setting the condition that the voltage difference between the two cut nodes gets to zero. This is the compensation currents method ([7]) that uses the reduced Thévenin impedance matrix and an array of known terms constituted by the open circuit voltage between two cut nodes; these are the voltages attained at the end of a forward phase. Since the condition imposed at the cut nodes is linear (voltages equality), the system to be solved is also linear and its solution goes through the inversion of the reduced Thévenin impedance matrix. The latter is composed of terms that do not depend on the bus voltages, therefore it is enough to invert it once and keep the coefficients of the inverse matrix to use them during the iterations.
The compensation current method is used also to solve systems with PV nodes ([8], [9], [10]); in this case, fictitious meshes, attained connecting a null impedance branch between the PV node and the voltage reference bus of the source node, are considered. In the null impedance branch an ideal voltage generator is included; its intensity is equal to the prefixed voltage module at the PV bus. In this case, the solution of the network is carried out with the described method, executing cuts in all the meshes; the cuts on the fictitious meshes are carried out so that the two cut nodes are: the PV node of the network, and the pole of the ideal voltage generator. The

construction of the reduced Thévenin impedance matrix is carried out based on all the meshes, both real and fictitious. As far as the known terms are concerned, namely the differences between the bus voltages at the cut nodes, there is a problem for the couple of nodes belonging to the fictitious meshes ([8], [9], [10],[11]). Indeed the voltage at the PV node is attained at the end of the forward phase on the radialized system and is given by a vector expressed referring to a general reference vector (normally the reference vector is the source node voltage expressed by a complex number with null imaginary part). Only the module, because prefixed, of the voltage of the ideal generator is known and the displacement is unknown. Different approaches have been suggested to solve this problem. In [8], in the two cut nodes for a PV node, the reactive power required to set to zero the voltage difference at the two nodes is calculated; the evaluation is carried out based on a sensitivity matrix that, under the hypothesis that the voltages have modules close to 1 p.u. and null reciprocal displacements, is the reduced Thévenin matrix. The same methodology is again considered and modified in [9] in order to accelerate the convergence; at the end of each iteration and before proceeding to the update of the loads currents, the bus voltages are corrected considering in the network only the compensation currents variations. In [10], the same methodology proposed in [9] is further developed with some modifications in the calculation of the compensation currents in the PV nodes; in this way, at each iteration, the current to be injected at the PV nodes is perpendicular to the bus voltage vector.
With reference to the above cited papers and to the relevant PV node model, the displacement of the ideal voltage generator is set equal to the displacement of the PV node voltage calculated at the end of a forward phase. This introduces a certain approximation in the calculation of the reactive power to be injected at the PV nodes. To eliminate such inconvenience, some methodologies imply that a correction of the attained results must be carried out at the end. This slows down the process to reach the final solution increasing the number of iterations.
In the methodology developed by the Authors in [11], the displacement of the ideal generator is summed up to the unknowns and is calculated imposing the condition that the complex power injected at the PV node is of reactive type.
Since such condition is non linear, in order to simplify the search of the solution, it is linearized under the hypothesis that the displacements between the voltages at the two ends of the cut nodes are limited.
Obviously, the linearization implies the introduction of a limited error in the final results related to the reactive power of the PV nodes.
In this paper the PV node is modelled by means of a shunt reactance. In the same way, at each iteration, the loads are represented by means of equivalent impedances calculated based on the bus voltage (fixed at the first iteration or calculated in the following iterations) and on the type of dependency of the load on the voltage. The meshes are opened in order to attain a radial system, and, at the cut nodes, the presence of compensation currents is considered. At each iteration, the solution process firstly requires the solution of the radial network with PV nodes and then the determination of the compensation currents. The radial system, made up only of impedances, is solved considering as unknowns, else than the susceptances of the PV nodes, the currents circulating on the impedances of the terminal nodes. The conditions imposed on the PV nodes voltage module and on the module and displacement of the voltage at the branching nodes and at the source node, allow, within the backward procedure, the determination one by one of all the unknowns of the network.
Once the bus voltages are determined, the compensation currents are immediately evaluated by means of a reduced Thévenin impedance matrix whose terms do not change from one iteration to the other. Before the verification of a generic convergence criterion, the bus voltages are corrected based on the values just calculated for the compensation currents.

2. General solution methodology for meshed systems with PV nodes

Consider a distribution system supplied by a source node at fixed voltage $\boldsymbol{V}_0$ having N nodes, M meshes and N_{PV} fixed voltage nodes. Executing for each mesh, a cut in one of the existing nodes, the total number of nodes becomes N+M; the two cut nodes at mesh k are indicated with T'_k and T''_k. Referring only to the cut nodes of the meshes, the real system can be schematized with the system shown in fig. 1. In the latter, for each of the cut nodes, a current is injected.

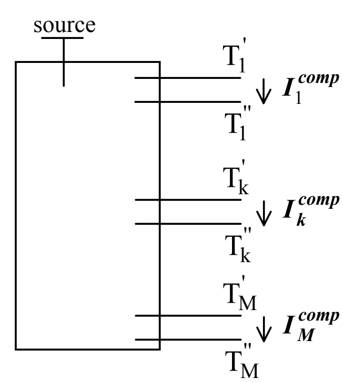

Fig. 1. Network scheme for the simulation of cut nodes.

The nodes T'_k are the cut nodes of the meshes in which the current is assumed to enter; the nodes T''_k are the cut nodes of the meshes in which the current is assumed to get out. Obviously, the currents injected in the cut nodes of the same mesh are equal and opposite ($\boldsymbol{I}'_k = -\boldsymbol{I}''_k = \boldsymbol{I}_k^{comp}$). All the compensation currents,

I_k^{comp}, are unknowns and can be evaluated by means of the reduced Thévenin impedance matrix and by means of the vector of the voltage differences between the two cut nodes. Indicating with $[\Delta V]$ the vector of the voltage differences at the two ends of the cut nodes and with [I^{comp}] the vector of the compensation currents injected in the cut nodes, it holds:

$$[\Delta V] = [Z] \cdot \left[I^{comp}\right] \tag{1}$$

where $[Z]$ is the reduced Thévenin impedance matrix. The self impedance term Z_{kk} related to the mesh k represents the summation of the branch impedances belonging to the path going from the node T'$_k$ to the node T''$_k$; the mutual impedance term Z_{ij} related to the meshes i and j represents the summation of the impedances of the branches that are common to the meshes i and j. Such summation is positive (negative) if the compensation current injected in node T'$_i$ determines in the cut nodes of the mesh j a voltage that has the same sign (opposite sign) with the positive sign fixed for the voltages between such two nodes. From (1), once the voltage differences between the cut nodes are known, the compensation currents can be deduced. The voltages at the cut nodes can be attained at each iteration, solving a radial system composed only of impedances. If in the system we have N_{PV} PV nodes and N_{Ter} terminal nodes, the unknowns are the N_{Ter} complex currents in the shunt impedances of the terminal nodes and the real susceptances of the N_{PV} PV nodes.

In the network are usually present branching nodes[1] (only for a main feeder without laterals they are not) and the source node. From the branching nodes one can attain N_{ter}-1 complex independent relations between the unknown currents. These can be attained by imposing the equality relation between the voltages at the branching nodes calculated based on the features related to a couple of branches spreading out from the node. The number of independent equations equals the number of branches spreading out from the node minus 1. From the source node one complex relation can be attained imposing that the voltage calculated equals the fixed one. For all the PV nodes one real relation can be attained imposing that the module of the bus voltage equals the prefixed value.

Therefore, from the mathematical point of view, the problem is solvable, nevertheless, the complete construction of the system of 2 N_{Ter} + N_{PV} real relations would give rise to a non linear system difficult to solve. On the contrary, the backward solution procedure set up allows the solution of all the equations. In this way, all the unknowns can be found using the conditions imposed at the branching nodes, in the PV node and in the source node. In this process, the elementary operation consists in the analysis of all the network branches following a suitable sequence and in the determination, branch by branch, of the voltage and current at the starting node based on the homonymous features at the arrival node.

Obviously the sequence chosen for the branches to be analyzed is such that a branch is analyzed only after all the other branches downstream are analysed.

Every time that, proceeding from the ending nodes towards the source node, a branching, a PV node or the source node is encountered, one or more unknowns are determined. In case of PV nodes the susceptance of the PV node is added as new unknown.

2.1 Solution of radial networks with PV nodes

The backward methodology set up analyses one branch at a time, starting from the terminal branches for which the shunt current is introduced as further unknown. The typology of the sending bus of the branch analysed identifies the type of unknown to be considered, namely to transfer to the branch upstream the unknowns on which branch voltages and currents depend. In what follows, the typical cases that can arise are examined.

2.1.1 Main feeder without laterals and PV nodes (fig. 2).

The current on the last shunt impedance, I_N, represents the only unknown of the network; indeed all the voltages and currents of the system are proportional to it. Therefore, going up to the source node, the features that each branch transfers to the branch upstream depend on the unknown current through a proportionality coefficient only depending on the networks impedances (that for the current iteration are constant). Imposing that the voltage at the source node equals the fixed value, it is possible to calculate the value of the unknown current and from the latter all the voltages and currents of the network:

$$V_{source} = H_{V,o} \cdot I_N \tag{2}$$

where the coefficient $H_{V,o}$ depends on the series and shunt impedances of the network in the path connecting the terminal node to the source node.

2.1.2 Main feeder with one lateral and without PV nodes (fig. 3).

The currents on the two terminal shunt impedances are unknown, I_{N1} and I_{N2}; proceeding from the two terminal branches and going towards the source node, the two branches having as sending bus the branching node, D, give for the voltage in the same node, two relations:

$$V_D = H_{V,D} \cdot I_{N1} \tag{3}$$

$$V'_D = H'_{V,D} \cdot I_{N2} \neq V_D \tag{4}$$

from which, imposing the equality constraint, a proportionality relation between the two unknown currents can be attained:

[1] Nodes from which spread out more than two branches.

$$I_{N2} = I_{N1} \cdot H_{V,D} \,/\, H'_{V,D} = I_{N1} \cdot k_D \tag{5}$$

Multiplying by k_D all the currents and voltages of the lateral, the same features are now proportional to the unknown current at the terminal node of the main feeder, I_{N1} (or vice versa), reducing to one the number of unknown currents.
The features on the branches upstream the branching node are, in this way, proportional only to one current and, from the analysis of the branch connected to the source node imposing that the voltage calculated equals the specified voltage, the unknown current can be attained and from it all the other features of the network:

$$V_{source} = H'_{V,o} \cdot I_{N1} \tag{6}$$

In this expression, the coefficient $H'_{V,o}$ depends on the series and shunt impedances of all the network (main feeder and lateral).

2.1.3 Main feeder without laterals and with 2 PV nodes (fig. 4).

The unknowns are: the current on the shunt terminal impedance (module and displacement) and the susceptances of the two PV nodes. Starting from the terminal branch, the voltages and currents of the branches downstream node 2 are proportional to the unknown current I_N. The voltage at the sending bus of the branch downstream node 2 is therefore expressed by:
$V_2 = H_{V,2} I_N e^{j\varphi}$
in which the module and displacement of the current I_N are unknown.
Being node 2 a PV node, the following condition must be satisfied:

$$|V_2| = |H_{V,2} I_N| = V_2^{sp} \tag{7}$$

which does not depend of the displacement of the current. Therefore from equation (7) the exact value of the module of the current I_N can be deduced and, with it, it is possible to attain the exact values of the modules of the voltages and currents of the branches downstream node 2. The branch current that is transferred upstream node 2 keeps then the displacement of I_N as unknown; in the branch upstream node 2 the presence of the reactive current due to the susceptance B_2 of the PV node must be considered; such current is given by:

$$I_{B_2} = j\, B_2 V_2 = j\, B_2 H_{V,2} I_N e^{j\varphi} \tag{8}$$

It depends on two unknowns: the displacement of current I_N and the susceptance B_2. Finally, the branch current upstream can be expressed as the summation of two components: one depending only on the displacement φ of I_N and the other depending on the displacement φ and on the susceptance B_2. The voltages and currents of all the branches between nodes 2 and 1, are given by the summation of two components of the above indicated type. In particular, the voltage at node 1 is expressed, as a function of the features of the branch downstream, by:

$$V_1 = H_{V,1} I_N e^{j\varphi} + B_2 H^B_{V,1} I_N e^{j\varphi} \tag{9}$$

whose module does not depend on the displacement of I_N, φ (unknown). Imposing that the module of V_1 equals the imposed value:

$$V_1^{imp} = | H_{V,1} I_N + B_2 H^B_{V,1} I_N | \tag{10}$$

from which:

$$\left(V_1^{imp}\right)^2 = [Re\left(H_{V,1} I_N\right) + B_2 Re\left(H^B_{V,1} I_N\right)]^2 + \\ + [Im\left(H_{V,1} I_N\right) + B_2 Im\left(H^B_{V,1} I_N\right)]^2 \tag{11}$$

which gives rise to the following second order equation in the unknown B_2:

$$aB_2^2 + 2bB_2 + c = 0 \tag{12}$$

where:

$$a = |\left(H^B_{V,1} I_N\right)|^2 \tag{13}$$

$$b = Re\left(H_{V,1} I_N\right) Re\left(H^B_{V,1} I_N\right) + Im\left(H_{V,1} I_N\right) Im\left(H^B_{V,1} I_N\right) \tag{14}$$

$$c = | H_{V,1} I_N |^2 - \left(V_1^{imp}\right)^2 \tag{15}$$

whose solution is:

$$B_2 = \frac{-b \pm \sqrt{b^2 - ac}}{a} \tag{16}$$

where the choice of the sign – allows to have higher values of susceptance and thus the lowest values of reactive power. Known the susceptance B_2 it is possible to correct all the features depending on B_2 in the path between nodes 1 and 2. It is thus eliminated one unknown, but in the branch upstream the node, the current circulating on the unknown admittance must be included, jB_1, associated to the node PV 1 (the displacement φ of the current I_N is still unknown).
Starting from the branch upstream node 1, all the voltages and currents can be expressed as the summation of two components: one depending on the displacement φ of I_N

and the other depending on φ and on the susceptance B_1. At the branch connected to the source node, imposing that the voltage calculated equals the imposed voltage:

$$\boldsymbol{V}_{source} = \boldsymbol{H}_{V,source} \boldsymbol{I}_N e^{j\varphi} + B_1 \boldsymbol{H}^B_{V,source} \boldsymbol{I}_N e^{j\varphi} \tag{17}$$

from the equality of the modules, the value of the susceptance B_1 can be deduced (starting from a II degree equation analogous to 12). Executed the correction of the components of voltage and current that depend on B_1 in the branches between the source node and node 1, imposing the equality of the displacements in (17) the value of φ can be deduced and all the voltages and currents of the network can be corrected.

2.1.4 Feeder with two laterals with two PV nodes (fig. 5).

Going from the terminal nodes up to the root node, the conditions imposed at the two PV nodes allow to deduce the correct value of the modules of the two shunt terminal currents. Therefore, in the branches upstream the two PV nodes, voltages and currents are expressed as the summation of two components; for the branches belonging to the lateral L_1, a component depends on the displacement φ_1 and the other on the displacement φ_1 and from the susceptance B_1. For the branches belonging to the lateral L_2, a component depends on the displacement φ_2 and the other on the displacement φ_2 and on the susceptance B_2. The voltage at the node D, based on the features of the branch belonging to the lateral L_1 and those of the branch belonging to the lateral L_2 is expressed by the two following equations:

$$\boldsymbol{V}_D = \boldsymbol{H}^1_{V,D} \boldsymbol{I}_{N,1} e^{j\varphi_1} + B_1 \boldsymbol{H}^{B_1}_{V,D} \boldsymbol{I}_{N,1} e^{j\varphi_1} \tag{18}$$

$$\boldsymbol{V}'_D = \boldsymbol{H}^2_{V,D} \boldsymbol{I}_{N,2} e^{j\varphi_2} + B_2 \boldsymbol{H}^{B_2}_{V,D} \boldsymbol{I}_{N,2} e^{j\varphi_2} \tag{19}$$

From the condition $\boldsymbol{V}_D = \boldsymbol{V}'_D$ two relations can be attained, one related to the equality of the modules and the other related to the equality of the displacements; from the first, considering that the modules do not depend on the unknown displacements, one of the two susceptances can be deduced, as an example, the B_2. The process is identical to that seen in the preceding paragraph with the only difference that in equation 15, instead than $\left(V_1^{imp}\right)^2$ we have $|\boldsymbol{V}_D|^2$; calculated the susceptance B_2. all the features depending on it can be corrected in the path between node D and node 2. Once the correction is executed, imposing the equality of the displacements, the correction of the displacement of all the features of the branches belonging to the lateral L_2 can be evaluated and corrected. In this way, the displacements of such features are corrected compared to the voltage of node D calculated based on φ_1 and B_1 still unknown. Note that, differently from the preceding case, the evaluation of one of the susceptances depends on the value initially assigned to the other and since the link is non linear, the final calculation of the unknown susceptance is approximated. Starting from the branch upstream node D all the voltages and currents can be expressed as the summation of two components: one depending only on the displacement φ_1 of $\boldsymbol{I}_{N,1}$ and the other depending on φ_1 and on the susceptance B_1. We are still in the conditions of the preceding case; therefore, at the branch connected to the source node, imposing that the voltage calculated equals, in module and displacement, to the imposed one, the last two unknowns can be deduced, B_1 and φ_1.

3. Implementation of the methodology

The evaluation of the following unknowns: the susceptances B in the PV nodes, and the currents $\boldsymbol{I}^{comp}$ between the cut nodes of the meshes, is executed calculating, at each iteration, the variations of such features compared to their values at the end of the preceding iteration. In other terms, the susceptance calculated at the end of one iteration is summed up to the one already existing in the PV node, and the current $\boldsymbol{I}^{comp}$ $(= \boldsymbol{I}_{T'} = -\boldsymbol{I}_{T''})$ is summed up to those already existing in the two cut nodes T' and T".

The main steps of the procedure are the following:

1. construction of the reduced Thévenin impedance matrix of the impedances between the cut nodes of the meshes. From the bus impedance matrix, by inversion, an admittance matrix can be attained, whose terms do not change in all the iterative process; the latter, multiplied by the vector of the voltage differences between two cut nodes of each mesh, allows to attain the compensation currents;
2. initialization of the bus voltages;
3. calculation of the equivalent loads impedances;
4. initialization of the values of the unknowns, currents in the shunt impedances of the terminal nodes and PV nodes susceptances;
5. following a fixed sequence, solution of each branch, namely calculation of the features at the sending bus based on the features values at the ending bus. If the sending bus is only a branching node or a PV node or the source node, calculation of one or two unknowns and correction of the features that depend of the just evaluated unknowns;
6. calculation of the compensation currents;
7. correction of the bus voltages of the network ([9]) as an effect of the currents injected in the cut nodes of the meshes;
8. comparison between the bus voltages values just calculated and those assumed by the same voltages at the beginning of the iteration; if the error is below a prefixed margin, the iterative process stops, otherwise another iteration is executed starting from step 3.

4. Application

The developed methodology was applied to solve the network with 85 nodes and 10 meshes shown in fig. 6; the data about lines and loads are reported in [2]. The presence of a given number of PV nodes was considered (ranging from 1 to 7) as well as the presence of a given number of meshes (from 1 to 10). Also two different loading conditions have been studied. The first is the operation with the rated loads, the second with all the loads increased of 50%. In table I, for two loading levels and for given numbers of PV nodes and meshes, the number of iterations required to reach the final solution are reported. The convergence factor ε is fixed equal to 10^{-4}; the module of the voltage at the PV nodes was fixed to 1 p.u. when the load is at the rated value and equal to 0.95 p.u. when the load is increased of 50%. For all the analysed cases also the reactive power injected at the PV nodes was calculated and compared to the value attained solving the system with the Newton-Raphson method. In all the examined cases, the absolute value of the difference between two values has the same order of magnitude of the convergence factor. The results indicate that the number of iteration grows with the loading and of the number of meshes and PV nodes. For the same number of PV nodes, the growth is fast for limited numbers of meshes (1,2); starting from 4 meshes the increase is null or limited. With the same number of meshes, the iterations required increase almost uniformly with the number of PV nodes. From the simplest condition, one mesh and one PV node, to the most complex, ten meshes and seven PV nodes, the number of iterations goes from 5 to 12 for a rated loading, and from 6 to 13 for loading increased of 50 %. The CPU time for iteration goes from 0.0105 s to 0.0114 s independently from the loading level.

5. Conclusion

The presence in distribution systems of fixed voltage nodes requires the setting up of analysis methodologies of such networks in which the simulation of the PV nodes gives reliable results with a limited number of iterations and with limited calculation times. The b/f methods now developed for the analysis of radial systems have been extended also to the case of meshed systems with PV nodes. The latter are considered with the creation of fictitious meshes. The network with real and fictitious meshes is then radialized by means of cuts and can thus be solved by the b/f technique. Once the voltages at the cut nodes are known, the unknown features can then be determined, these being associated both to the meshes and to the PV nodes.

The solution technique here developed (and then extended to meshed systems) requires the schematization at each iteration of the loads by means of equivalent impedances; the network is composed only of series and shunt impedances and is supplied by only one node. For the solution of such type of network a backward technique has been set up in which are considered unknown the currents in the shunt impedances of the terminal buses. Proceeding from the terminal branches up to the root node, the condition imposed at the voltage of the branching nodes and of the source node allow the elimination, one by one, of all the unknowns. The PV node model here proposed implies its schematization by means of a susceptance, therefore, if in the radial system there are PV nodes to the cited unknowns also the PV nodes susceptances must be added.

In a similar way, analyzing the branches from the terminal nodes up to the root node, the conditions imposed to the module of the voltage at the PV node and those imposed for the voltage of the branching nodes and of the source node, allow the evaluation one by one of all the unknowns. After having solved the radialized system, keeping into account the PV nodes, the compensation currents, that take into account the presence of meshes, can be considered..

The results of the applications have shown that the number of iterations is limited also when highly meshed systems with many PV nodes and heavily loaded must be solved. The precision of the results is the same as that of the results that can be attained with the techniques usually adopted for transmission systems.

Table I – Network with 85 nodes and 10 meshes.
Number of iterations, for two loading levels, as the number of PV nodes and meshes varies.

	Rated load						**Rated load increased of 50 %**					
Meshes number	1	2	4	6	8	10	1	2	4	6	8	10
PV nodes number												
1	5	6	9	9	9	9	6	7	10	10	10	10
2	5	7	11	11	11	11	6	8	10	10	10	10
3	6	7	11	11	11	11	6	8	10	10	10	10
4	6	7	11	11	11	11	7	8	10	10	10	10
5	7	8	10	10	10	10	8	8	10	10	12	12
6	9	9	10	10	10	10	10	11	12	12	13	13
7	9	9	11	11	12	12	10	11	12	12	13	13

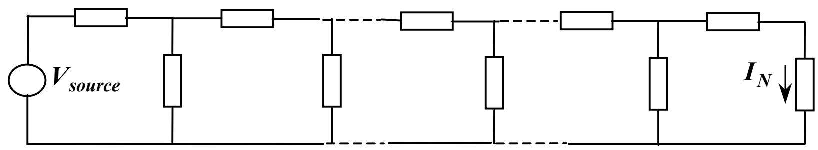

Fig. 2. Circuital model made of N four-pole networks connected in cascade.

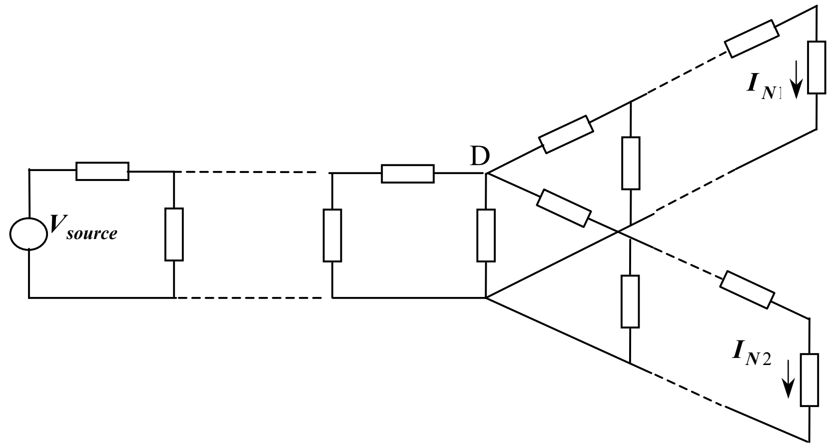

Fig. 3. Circuital model of a simple network with a branching point at node D.

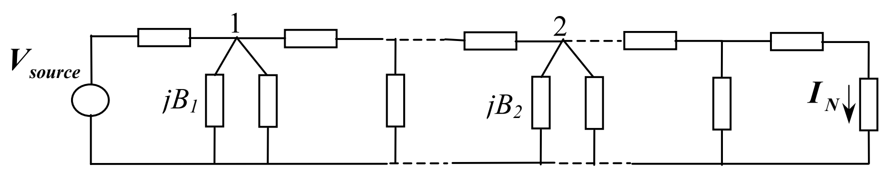

Fig. 4. Circuital model of a network with two PV nodes.

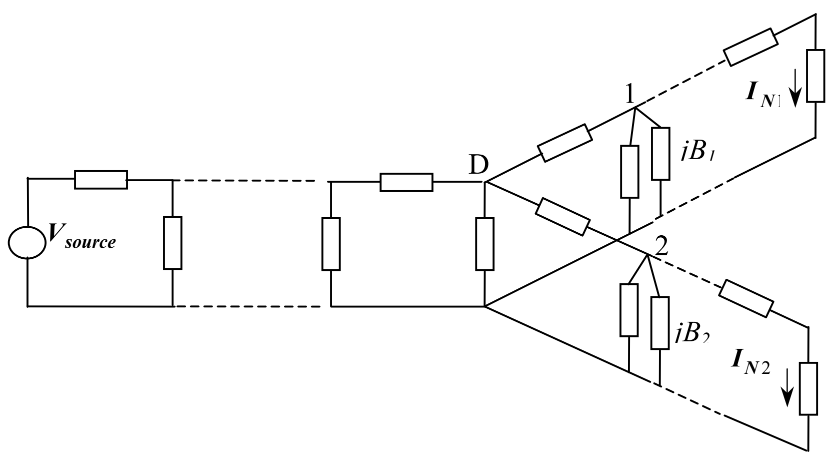

Fig. 5. Circuital model of a network with a branching point (node D) and two PV nodes (nodes 1 and 2).

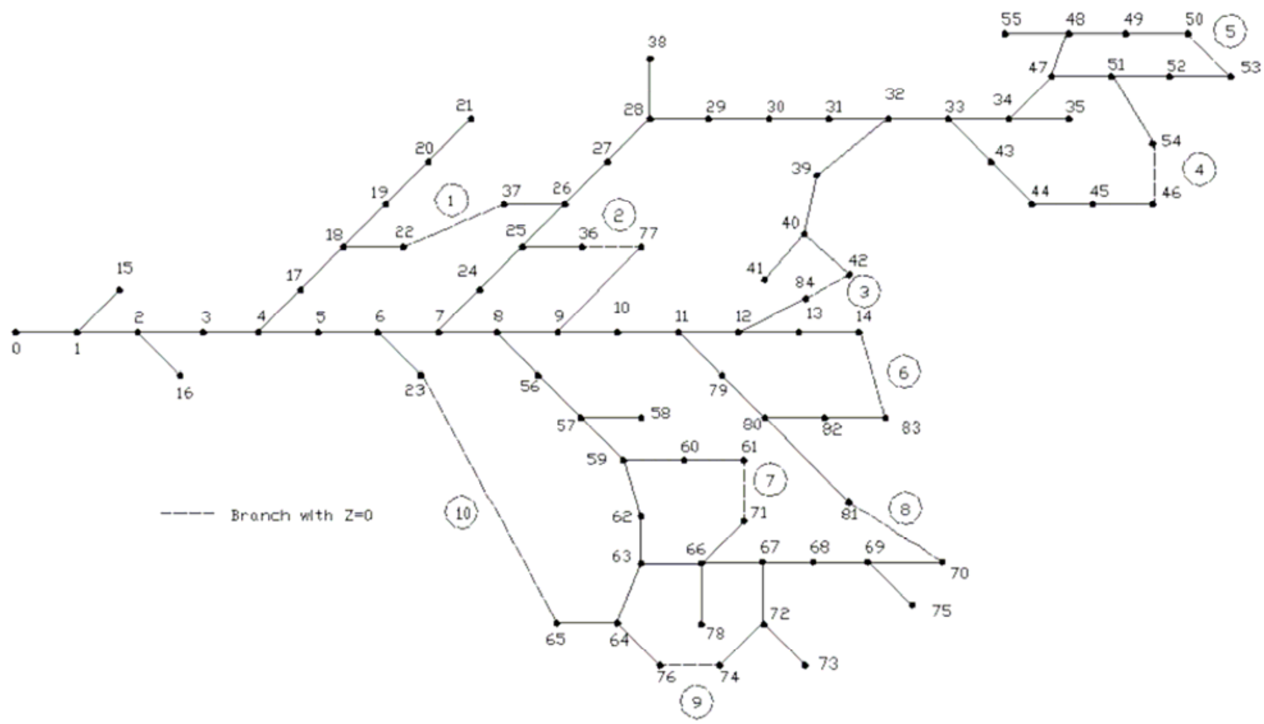

Fig. 6. Test system with 85 nodes and 10 meshes.

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