Proceedings of the Seventh IASTED International Conference
POWER AND ENERGY SYSTEMS
August 29–31, 2007, Palma de Mallorca, Spain
ISBN Hardcopy: 978-0-88986-689-8 / CD: 978-0-88986-690-4

EVT IN ELECTRICITY PRICE MODELING: EXTREME VALUE THEORY NOT ONLY ON THE EXTREME EVENTS

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ABSTRACT
Extreme value theory (EVT) is used to model extreme electricity prices in the literature. In this paper I show that EVT can be used to describe the distribution of the prices itself. I build a model on the price formation in electricity auctions markets. The conclusion is that the distribution of electricity prices should be a generalized extreme value (GEV) distribution. Empirical data confirm this conclusion.

KEY WORDS
Risk analysis, extreme value theory, electricity prices

1. Introduction

Extreme value theory (EVT) is used both in electricity and in financial risk modeling. Two major model types are used in risk modeling. The first is the block maxima method which central theorem is the Fisher-Tippett Theorem (see Section 2.1), and it is used to model the distribution of the maxima. The other EVT model is the peaks-over-threshold (POT) model which describes the distribution of values which are above a certain threshold. In electricity risk management the papers which cite EVT usually use it to model the right (and/or left) side of the distribution to characterize the extreme losses. [1] models the tails of the distribution using the POT method. [2] applies the POT method to calculate the value at risk of an electricity portfolio. [3] estimates value at risk using POT estimates.

I state that the EVT can also be used to model the electricity prices itself so electricity prices have a GEV distribution. It is so because the price in an auctions market is the maximum of the sellers' bids which are met and fulfilled. Therefore EVT applies for the price (being a maximum itself).

The model is similar to the result of [4] who derives the distribution of the expected highest valuation and the expected payoff in the case of a second-price auction. They apply their results to Internet auctions.

My model is an unconditional model according to [1]. This means that the data are not filtered before the analysis.

In the literature there are three main modeling approaches to describe the distribution of electricity prices. The models in the first model family fit a stochastic model on the electricity price time series and use this stochastic model to generate the given distribution (see [5]). The models of the second group consist fundamental descriptions of electricity demand and supply, and the driving factors of supply and demand determine the price distribution (see [6]). The third type of models is agent-based models which use simulation techniques to write down the price distribution (see [7]). This paper contains a fourth way of research. It aims to determine the distribution of electricity prices directly without knowing anything about the data generating process or market driving forces.

The structure of the paper is as follows: In section 2 I present a model of electricity price formation which concludes that the electricity prices shall have a generalized extreme value (GEV) distribution. Section 3 contains the empirical tests. Section 4 concludes the paper.

2. The model

2.1 Generalized extreme value distributions

Generalized extreme value (GEV) distributions have a cumulative distribution function (cdf) of the following form:

$$F_{k,\mu,\sigma}(x) = \exp\left\{-\left[1+k\left(\frac{x-\mu}{\sigma}\right)\right]^{-1/k}\right\}$$

if

$$1+k(x-\mu)/\sigma > 0$$

This is a cdf with three parameters: k is the shape parameter, sigma (>0) is the scale parameter, and mu is the location parameter. GEV distribution family is a special case of three distributions. We can speak of Fréchet distribution if k>0, of Weibull distribution if k<0, and of Gumbel distribution if k→0.

The GEV distribution is a central distribution in EVT according to the Fisher-Tippett Theorem. This theorem states the following (see [8]):

Let x_1, x_2,…, x_n be independently and identically distributed (IID) random variables. Let M_n be the maximum of these random variables.

If there exist sequences of real constants c_n(>0) and d_n that

$$\frac{M_n - d_n}{c_n} \xrightarrow{dist} H$$

for some nondegenerate H then H must be a GEV distribution

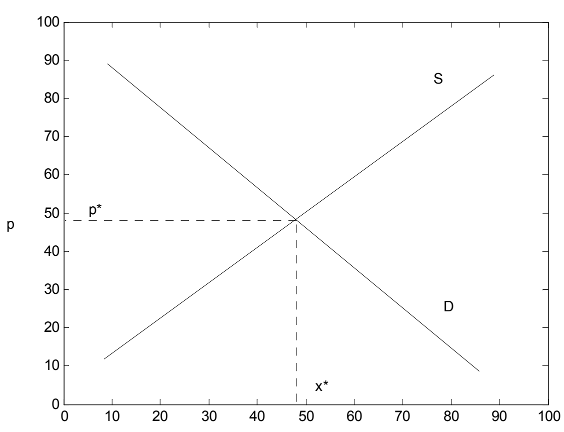
The type of the limiting distribution is unique with respect to the shape parameter of the GEV distribution.

So the limiting distribution of the appropriately transformed block maxima is a GEV distribution.

2.2 Electricity auctions markets

Electricity is traded in different markets. One of these markets is the "day-ahead" market at which participants trade electricity to be distributed the following day. A widely used market mechanism for day-ahead markets is trading via an auction. In these markets electricity sellers and buyers submit their bids or bid curves to the exchange. Then a market supply and demand curve is constructed. The price will be the market clearing price at which the demand and supply are equal as illustrated in Figure 1. In reality supply and demand functions are not continuous.

Figure 1 Illustration to price formation in electricity auctions markets



All the sellers' bids are met which are lower than the market-clearing price, and all the buyers' bids are fulfilled which are higher than the market clearing price. To put it another way, the market clearing price is the maximum of the sellers' bids which are met, and the minimum of the buyers' bids which are met.

2.3 The model

Because the price of electricity (the market clearing price) is the maximum of the seller's bids its distribution is the extreme value distribution of the seller's bids. Therefore electricity prices should have an extreme value distribution of some type. The specific form of the distribution depends on the distribution of the sellers' bids and the way how these bids are related to each other.

If we assume that the seller's bids are independently and identically distributed (IID) then the bids are subject to the Fisher-Tippett Theorem. As the limiting distribution in the Fisher-Tippett Theorem is unique, the limiting distribution has to be the same as the distribution of the price. So the price distribution has to be a GEV distribution.

The first assumption to conclude this is that sellers' bids have to be IID. The second is that we have to forget about the price formation mechanism and have to deal only with the distribution of sellers' bids. We don't have to forget that in calculating the extreme value distribution of the seller' bids we should take only those bids into account which are met.

This is acceptable if we assume that f() is the probability density function of the sellers' bids which are met. We select a sample from this distribution. We consider this sample as the bids which are accepted. There are bids which are not in the sample; the reason for this is not known. It can happen that these bids were too high to be met, or these bids were not present in the market (for example the power plant which would make this bid had an outage). We are not sure why a specific bid is out of sample but we can be sure that the probability of this bid to be out of sample will work out according to f(). The reason for this is the fact that f() has all the information about the probability for a certain bid to get into the sample.

This explains that we can forget about the market mechanism because f() has already all the information from it. We choose our sample from f() so the bids will be identically distributed.

The strongest assumption is that the bids are independent. If the bids are dependent then if we choose a bid then we tend to choose another one. Or, if one bid is out-of-sample then the other one is also out-of-sample. This can happen if a power plant usually works with total capacity or doesn't work at all. Then the bids of this factory can't be considered independent. The assumption of independency can only hold if we exclude this kind of behavior.

Assuming independency also contains assuming lack of strategic interaction between the firms. So the bid of one firm has to be independent from the bid of another firm.

One may think that the price distribution depends only on the factors affecting the supply bids. On the contrary, it also incorporates the factors affecting the demand side. All these random factors are taken into account in f().

The results of the Fisher-Tippett Theorem can be applied when the sample has many elements. As the number of bids chosen is the traded quantity then the former arguments are valid if the traded quantity is high.
To sum it all up: when the traded quantity is high then electricity prices have GEV distribution if we assume that the bids are independent. This assumption means lack of strategic interaction and that we exclude certain type of outages or behavior.

Next I test the results using empirical data.

3. Empirical distributions

3.1 Data and methodology

I investigated two markets: the EEX spot market and the APX day-ahead market. Both markets use auctions to determine the market-clearing hourly electricity price. I have used data from June 2000 to April 2007 in the case of EEX and data from January 2002 to December 2003 in the case of APX. I calculated the daily prices as the sum of the hourly prices and used these data for analysis.

Figure 2 shows time series daily electricity prices for the two markets. Price strikes can be identified in both cases.
The calculations were made in MATLAB. I have used MATLAB Distribution Fitting Tool to estimate the parameters of the distributions. I also calculated chi-squared goodness-of-fit tests and used Q-Q plots.

3.2 Results

The estimated parameters of the GEV distributions are shown in Table 1. EEX daily data have a shape parameter of 0.124025 therefore they have a Fréchet distribution. APX daily data have a shape parameter of 0.265731 therefore they also have a Fréchet distribution.

Table 1 Estimated GEV parameters

Parameter	Estimate (EEX)	Estimate(APX)
k	0.124025	0.265731
mu	586.806	584.86
sigma	258.382	261.849

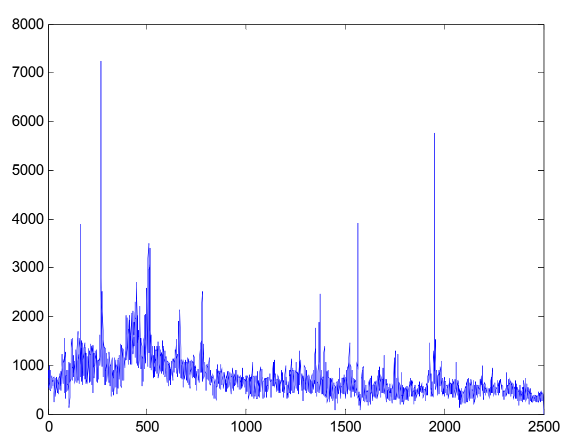
Figure 3 presents the distribution graph for empirical and fitted data. It can be seen that the estimated probability density function (pdf) fits the empirical probabilities very well.

Figure 4 illustrates the same for the cumulative distribution function (cdf). According to these graphs the GEV distribution is a good estimate of the empirical cdf.
Figure 5 presents the Q-Q plots. They show the quantiles of the empirical and the estimated distributions. These graphs prove the conclusion that electricity prices have a GEV distribution.

I also performed a numerical statistical test. Table 2 has the chi-squared statistics and the related p-values. The null hypothesis of this test is that the APX and EEX prices have a GEV distribution. The p-values show that we can't reject the null hypothesis (under the usual significance levels). Therefore we can say that APX and EEX prices have GEV distribution.

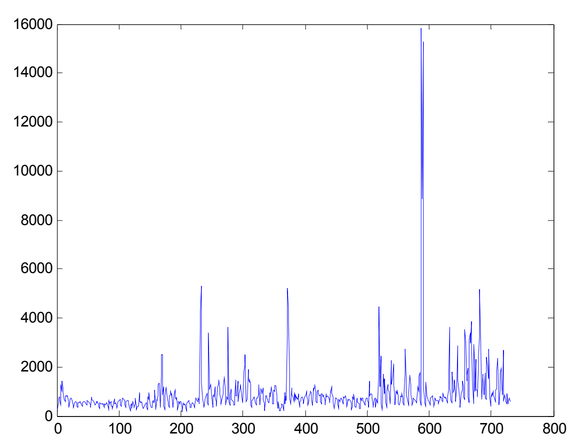
Figure 2: EEX and APX time series data (daily prices)

EEX price time series



Data: www.eex.com

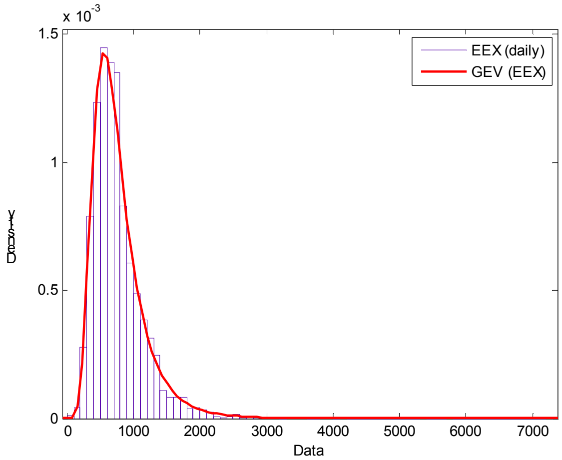
APX price time series



Data: www.apx.nl

Figure 3: Empirical and estimated GEV pdf for EEX and APX daily data

EEX price distribution



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APX price distribution

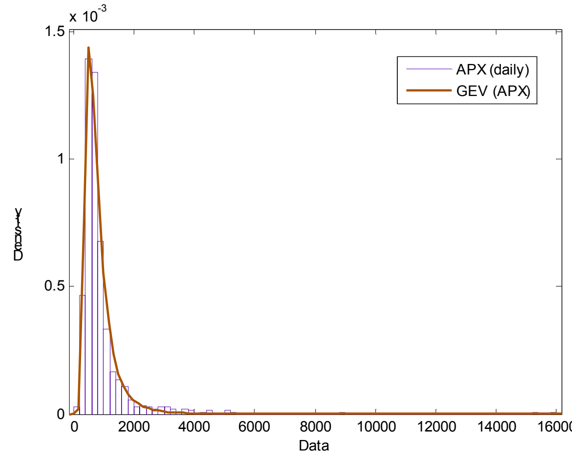

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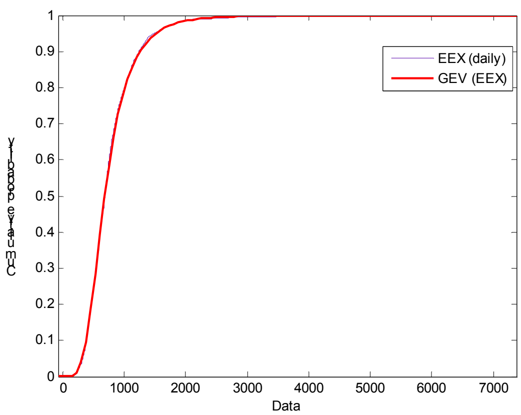
Figure 4: Empirical, estimated GEV, and estimated lognormal cdf for EEX hourly data

EEX price distribution



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APX price distribution

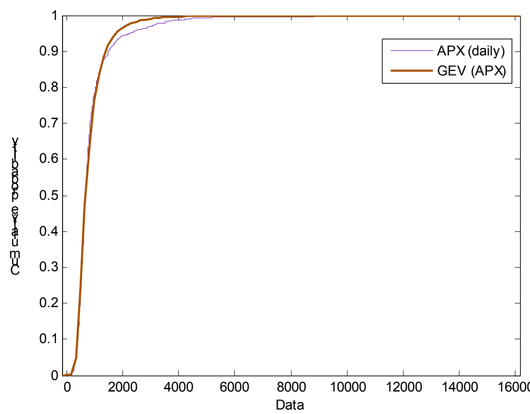

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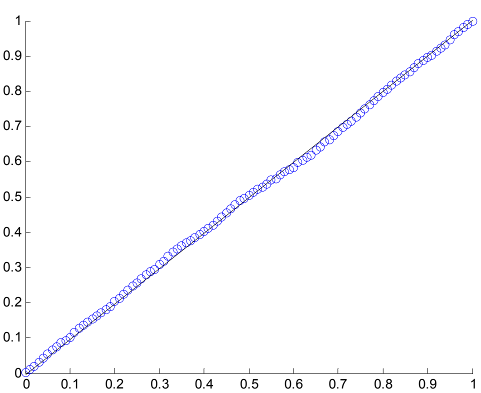
Figure 5: Q-Q plots (empirical daily prices versus estimated GEV distributions)

EEX Q-Q plot



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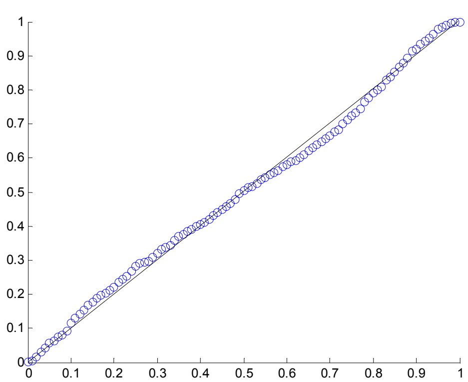
APX Q-Q plot



Data: www.apx.nl

Table 2: Chi-squared statistics and p-values

	Chi-squared statistics	p-value
APX	39.6287	0.8877
EEX	141.8725	0.9250

The graphs and statistical test confirm the model.

4. Conclusion

Electricity prices in auctions markets are the maximum of those bids which are met. If we assume that these bids are independent then we can apply the Fisher-Tippett Theorem, and say that electricity prices have GEV distribution.

Empirical data confirm the assumption of GEV distribution. The fitted distributions were Fréchet type distributions. It is subject to future research whether it is a general rule for electricity markets or not.

Future research has to expose the bid interactions which can be assumed to meet the empirical distributions. We may weaken the assumption of independency.

References

[1] H. Byström, Extreme value theory and extremely large electricity price changes. Working paper, Department of Economics, Lund University, 2001.

[2] R. Gencay & F. Selcuk, Extreme value theory and value at risk: Relative performance in emerging markets, Working paper, University of Windsor, Ontario, Canada, 2002.

[3] R. de Rozario, Estimating value at risk for the electricity market using a technique from extreme value theory. Working paper, School of Banking and Finance, University of NSW, AU, 2002.

[4] S. Caserta & C.G. de Vries, Auctions with numerous bidders. Working paper, 2004.

[5] N. Haldrup & M.O. Nielsen, Directional congestion and regime switching in a long memory model for electricity prices, *Studies in Nonlinear Dynamics & Econometrics, 10*(3), 2006, Article 1.

[6] A. Eydeland & K. Wolyniec, *Energy and power risk management. New developments in modeling, pricing, and hedging* (Hoboken, New Jersey, John Wiley & Sons, 2003).

[7] I. Praca, C. Ramos, Z. Vale & M. Cordeiro, A new agent-based framework for the simulation of electricity markets. *Proceedings of the IEEE/WIC International Conference on Intelligent Agent Technology*, 2003. P469.

[8] A.J. McNeil, R. Frey & P. Embrechts, *Quantitative risk management. Concepts, techniques, tools* (New Jersey, Princeton Series in Finance, 2005).