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HEAD-DEPENDENT MAXIMUM POWER GENERATION IN SHORT-TERM HYDRO SCHEDULING USING NONLINEAR PROGRAMMING

Sílvio J. P. S. Mariano, João P. S. Catalão
Universidade da Beira Interior
R. Fonte do Lameiro
6201-001 Covilhã, Portugal
sm@ubi.pt; catalao@ubi.pt

Victor M. F. Mendes
Instituto Superior de Engenharia
R. Conselheiro Emídio Navarro
1950-062 Lisboa, Portugal
vfmendes@isel.pt

Luís A. F. M. Ferreira
Instituto Superior Técnico
Av. Rovisco Pais
1049-001 Lisboa, Portugal
lmf@ist.utl.pt

ABSTRACT
This paper is on the problem of short-term hydro scheduling. We propose a method, based on nonlinear programming, which considers hydroelectric power generation as a nonlinear function of water discharge and of the head. The main contribution of this paper is that the maximum power generation is also considered as head-dependent in order to obtain more realistic and feasible results. The proposed method has been applied successfully to solve a case study based on one of the Portuguese cascaded hydro systems, providing a higher profit at an acceptable computation time in comparison with classical optimization methods based on linear programming that ignore head dependence.

KEY WORDS
Short-term hydro scheduling, nonlinear programming, power generation, and head dependence

1. Introduction

The satisfaction of the demand for electric energy has been mainly achieved with hydro resources and thermal resources. Hydro resources particularly run-of-the river resources are considered to provide a clean and environmentally friendly energy option, while thermal resources particularly fossil fuel-based resources are considered to provide an environmentally aggressive energy option, but nevertheless still in nowadays a necessary option. Hence, promoting efficiency improvements in the exploitation of the hydro resources is increasingly important, reducing the reliance on fossil fuels and decreasing greenhouse emissions which are major contributors to climate change.

The hydro scheduling problem is usually divided into different time horizons: short, medium and long-term hydro scheduling. Short-term hydro scheduling (STHS) encircles a time horizon of one day to one week, usually divided in hourly intervals. Deterministic models are used. Where stochastic quantities are included, such as hydro inflows or energy prices, the corresponding forecasts are used [1].

In a deregulated profit-based environment, such as the Norwegian case [2] or concerning Portugal and Spain given the forthcoming Iberian Electricity Market (MIBEL), the optimal management of the water available in the reservoirs for power generation, without affecting future operation use, represents a major advantage for generating companies (GENCOs) to face competitiveness given the economic stakes involved.

The main goal in the profit-based hydro scheduling problem is to maximize the value of total hydroelectric generation throughout the time horizon, while satisfying all hydraulic constraints, aiming the most efficient and profitable use of the water [3]. Hence, the improvement of existing hydro scheduling models promoting a better exploitation efficiency of hydro resources is an important line of research [4], especially for head-dependent reservoirs in light of market conditions [5].

In hydro plants with a small storage capacity available, also known as run-of-the-river hydro plants, the power generation efficiency becomes sensitive to the head — head change effect. For instance, in the Portuguese system there are several hydro chains formed by many but small reservoirs. Hence, it is necessary to consider the head change effect on STHS in order to obtain more realistic and feasible results.

Dynamic programming (DP) is among the earliest methods applied to the STHS problem [6]. Although, DP can handle the non-concavities and the nonlinear characteristics present in the hydro model, direct application of DP methods for hydro systems with many coupled plants is impractical due to the well-known DP curse of dimensionality, more difficult to avoid in short-term than in long-term optimization without losing the accuracy needed in the model [7].

Artificial intelligence techniques have also been applied to the STHS problem, namely, neural networks [8] and genetic algorithms [9]. However, a large computational effort is necessary to solve the problem for a time horizon of one week. Also, due to the heuristics used in the search process optimal solutions are difficult.

A natural approach is to model the system as a network flow model, because of the underlying network structure subjacent in cascaded reservoirs [10]. The network flow model is often simplified to a linear or piecewise linear one. Linear programming (LP) is a widely used method for STHS [11]. LP algorithms lead to extremely efficient codes, implementations of which can be found commercially. However, LP algorithms imply that power generation is linearly dependent on water discharge, thus neglecting head dependence to avoid nonlinearities, leading to inaccuracy.

Mixed-integer linear programming (MILP) is becoming frequently used for STHS [12], where binary variables allow modeling of start-up costs, which are mainly due to wear and tear of the windings and to malfunctions of the control equipment. However, the discretization of the nonlinear dependence between power generation, water discharge and head, used in MILP to model head variations, augment the computational burden required to solve this problem.

This paper proposes a nonlinear programming (NLP) method to solve the STHS problem considering head dependence. This method expresses hydro generation characteristics more accurately and the head change effect can be taken into account [13]. The nonlinear dependence between the power generation, the water discharge and the head is taken into account in our study through a nonlinear formulation, which represents one of the main difficulties associated with the STHS problem. In an earlier formulation [14], the maximum power generation was considered constant. As a new contribution, the maximum power generation is also considered head-dependent in this formulation, in order to obtain more realistic and feasible results.

2. Notation

The notation used throughout the paper is described below.

K	Total number of hours in the scheduling time horizon
J	Total number of hydro resources
l_{kj}	Water level in reservoir j during period k
$l_j^{\max}$	Maximum water level in reservoir j
$l_j^{\min}$	Minimum water level in reservoir j
h_{kj}	Head of plant j during period k
$h_j^{\max}$	Maximum head of plant j
$h_j^{\min}$	Minimum head of plant j
v_{kj}	Water storage of reservoir j at end of period k
$v_j^{\max}$	Maximum storage of reservoir j
$v_j^{\min}$	Minimum storage of reservoir j
v_{0j}	Initial water storage of reservoir j
v_{Kj}	Final water storage of reservoir j
q_{kj}	Water discharge of plant j during the period k
$q_{kj}^{\max}$	Maximum water discharge of plant j during the period k
$q_j^{\min}$	Minimum water discharge of plant j
s_{kj}	Water spillage by reservoir j during the period k
a_{kj}	Natural inflow to reservoir j during the period k
p_{kj}	Power generation of plant j during period k
η_{kj}	Efficiency of plant j during period k
$\eta_j^{\max}$	Maximum efficiency of plant j
$\eta_j^{\min}$	Minimum efficiency of plant j
τ_{mj}	Water travel delay between reservoirs m and j
λ_k	Forecasted energy price during period k
Ψ_j	Future value of the water stored in reservoir j
M_j	Set of upstream reservoirs to reservoir j
$\boldsymbol{F}$	Nonlinear function of variables
$\boldsymbol{A}$	Constraint matrix
$\boldsymbol{b}^{\max}$	Upper bound vector on constraints
$\boldsymbol{b}^{\min}$	Lower bound vector on constraints
$\boldsymbol{x}$	Vector of the flux variables corresponding to the arcs of the network
$\boldsymbol{x}^{\max}$	Upper bound vector on variables
$\boldsymbol{x}^{\min}$	Lower bound vector on variables

3. Formulation

The STHS problem is formulated as a NLP problem. The objective function to be maximized can be expressed as:

$$F = \sum_{k=1}^{K} \sum_{j=1}^{J} \lambda_k \, p_{kj} + \sum_{j=1}^{J} \Psi_j \, (v_{Kj}) \qquad (1)$$

The objective function in (1) is composed of two terms. The first term represents the profit with the hydro system during the short-term time horizon. The last term expresses the water value for the future use of the water stored in the reservoirs at the last period.

The optimal value of the objective function is determined subject to constraints: equality constraints and inequality constraints or simple bounds on the variables.

The following equations represent the set of constraints for the plants over the short-term time horizon.

- Water balance equation

$$v_{kj} = v_{k-1,j} + a_{kj} + \sum_{m \in M_j} (q_{k-\tau_{mj},m} + s_{k-\tau_{mj},m}) - q_{kj} - s_{kj} \quad (2)$$

- Power generation equation

$$p_{kj} = q_{kj}\, \eta_{kj}\, (h_{kj}) \quad (3)$$

- Head equation

$$h_{kj} = l_{kf(j)}\, (v_{kf(j)}) - l_{kt(j)}\, (v_{kt(j)}) \quad (4)$$

- Water storage constraints

$$v_j^{\min} \le v_{kj} \le v_j^{\max} \quad (5)$$

- Water discharge constraints

$$q_j^{\min} \le q_{kj} \le q_{kj}^{\max}\, (h_{kj}) \quad (6)$$

- Water spillage constraints

$$s_{kj} \ge 0 \quad (7)$$

Equation (2) corresponds to the water conservation equation. The travel time between reservoirs must be taken into account if the transportation delays are not negligible. In (3) power generation is considered a function of water discharge and of efficiency, expressed as the output-input ratio, which in turn depends on the head. In (4) the head is considered a function of the water level in the upstream reservoir and of the water level in the downstream reservoir, both levels depending on the water storages in the respectively reservoirs. In (5) water storage has lower and upper bounds. In (6) water discharge has lower and upper bounds. The minimum water discharge in our case study is considered null, but may be required to be non-zero due to navigation, recreational or ecological reasons. As a new contribution to earlier studies, the maximum water discharge is considered a function of the head. In (7) a null lower bound is considered for water spillage. Water spillage by the reservoirs can occur only in normal schedule situations when without it the water storage exceeds its upper bound, so spilling is necessary due to safety considerations. The initial water storages and the inflows to reservoirs are assumed as known input data.

4. NLP Method

In order to solve the STHS problem, it is essential to use appropriate models, considering power generation as a function of water discharge and also of the head for run-of-the-river hydro plants. This function is represented by the unit performance curves (Fig.1), a family of nonlinear curves each for a specified value of the head.

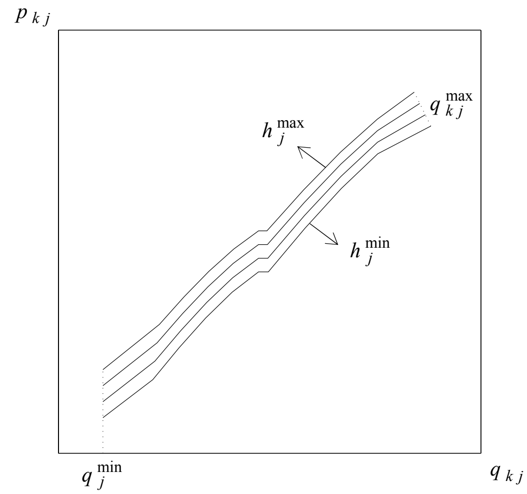

Fig.1 – Unit performance curves.

The main contribution of this paper is that the maximum water discharge is considered a function of the head as shown in (6), implying that the maximum power generation is also head-dependent. This is indicated by the dashed line labeled $q_{kj}^{\max}$ in Fig.1.

The STHS problem can be formulated as the following nonlinear optimisation problem:

$$Max \quad \boldsymbol{F}(\boldsymbol{x}) \quad (8)$$

$$subject\ to \quad \boldsymbol{b}^{\min} \le \boldsymbol{A}\,\boldsymbol{x} \le \boldsymbol{b}^{\max} \quad (9)$$

$$\boldsymbol{x}^{\min} \le \boldsymbol{x} \le \boldsymbol{x}^{\max} \quad (10)$$

The water balance in (2) is rewritten as in (9), setting the lower bound equal to the upper bound. The bounds on water storage, water discharge and water spillage in (5), (6) and (7), respectively, are rewritten as in the inequality constraints in (10). Also, due to the maximum water discharge head dependency, the upper bound on water discharge implies a new inequality constraint that will be rewritten as in (9).

In (3) the efficiency depends on the head. We consider it given by:

$$\eta_{kj} = \eta_j^0 + \alpha_j\, h_{kj} \quad (11)$$

where the parameters η_j^0 and α_j are respectively the offset and the slope given by:

$$\eta_j^0 = \eta_j^{\max} - \alpha_j\, h_j^{\max} \quad (12)$$

$$\alpha_j = (\eta_j^{\max} - \eta_j^{\min}) \,/\, (h_j^{\max} - h_j^{\min}) \quad (13)$$

In (4) the water level depends on the water storage. We assume it given by:

$$l_{kj} = l_j^0 + \beta_j \, v_{kj} \tag{14}$$

where the parameters l_j^0 and β_j are respectively the offset and the slope given by:

$$l_j^0 = l_j^{\max} - \beta_j \, v_j^{\max} \tag{15}$$

$$\beta_j = (l_j^{\max} - l_j^{\min}) \, / \, (v_j^{\max} - v_j^{\min}) \tag{16}$$

this assumption implies reservoirs with vertical walls, which is a good approximation for run-of-the-river reservoirs, due to its small storage capacity, as the data have shown for the case study.

Substituting (11) into (3) we have:

$$p_{kj} = q_{kj} \, (\eta_j^0 + \alpha_j \, h_{kj}) \tag{17}$$

By substituting (4) and (14) into (17) power generation becomes a nonlinear function of water discharge and water storage, given by:

$$\begin{aligned} p_{kj} = \eta_j^0 \, q_{kj} + \alpha_j \, l_{f(j)}^0 \, q_{kj} - \alpha_j \, l_{t(j)}^0 \, q_{kj} + \\ + \alpha_j \, \beta_{f(j)} \, q_{kj} \, v_{kf(j)} - \alpha_j \, \beta_{t(j)} \, q_{kj} \, v_{kt(j)} \end{aligned} \tag{18}$$

In our model, the maximum water discharge is considered head-dependent and it is given by:

$$q_{kj}^{\max} = q_j^0 + \delta_j \, h_{kj} \tag{19}$$

where the parameters q_j^0 and δ_j are respectively given by:

$$q_j^0 = q_j^1 - \delta_j \, h_j^{\max} \tag{20}$$

$$\delta_j = (q_j^1 - q_j^{\max}) \, / \, (h_j^{\max} - h_j^{\min}) \tag{21}$$

Substituting (4) and (14) into (19) we have:

$$\begin{aligned} q_{kj}^{\max} = q_j^0 + \delta_j \, l_{f(j)}^0 - \delta_j \, l_{t(j)}^0 + \\ + \delta_j \, \beta_{f(j)} \, v_{kf(j)} - \delta_j \, \beta_{t(j)} \, v_{kt(j)} \end{aligned} \tag{22}$$

The maximum water discharge becomes a function of water storage, given by:

$$q_{kj}^{\max} = \gamma_j^0 + \gamma_j^1 \, v_{kf(j)} - \gamma_j^2 \, v_{kt(j)} \tag{23}$$

where the parameters γ_j^0, γ_j^1 and γ_j^2 are respectively given by:

$$\gamma_j^0 = q_j^0 + \delta_j \, l_{f(j)}^0 - \delta_j \, l_{t(j)}^0 \tag{24}$$

$$\gamma_j^1 = \delta_j \, \beta_{f(j)} \tag{25}$$

$$\gamma_j^2 = \delta_j \, \beta_{t(j)} \tag{26}$$

Hence, the new inequality constraint to be rewritten as in (9) is given by:

$$q_{kj} - \gamma_j^1 \, v_{kf(j)} + \gamma_j^2 \, v_{kt(j)} \leq \gamma_j^0 \tag{27}$$

5. Case Study

The proposed NLP method has been applied on a case study based on one of the Portuguese hydro systems, consisting of three head-sensitive cascaded reservoirs. The spatial coupling among reservoirs is shown in Fig.2.

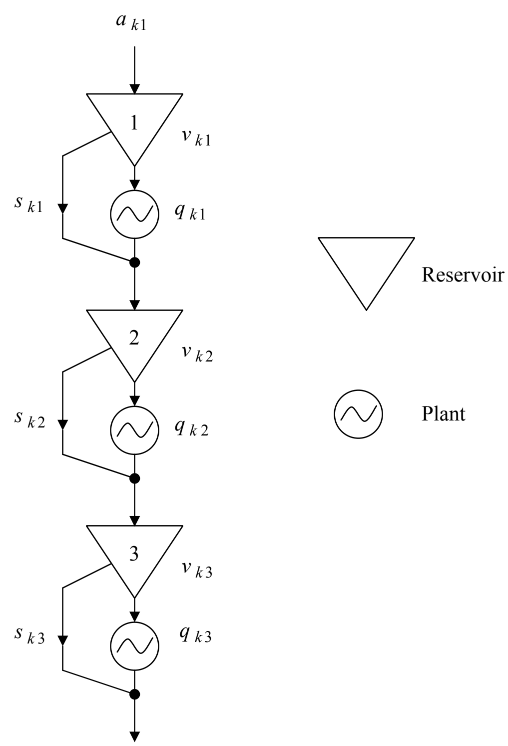

Fig.2 – Hydro system with three cascaded reservoirs.

Only the first reservoir has inflow. This inflow is due to an upstream watershed belonging to a different company and is shown in Fig.3.

Our model was implemented on a 600-MHz-based processor with 256 MB of RAM using the optimization solver package MINOS under MATLAB. The scheduling time horizon chosen is one week divided into 168 hourly periods.

The energy price profile over the time horizon is shown in Fig.4 (where $ is a symbolic economic quantity).

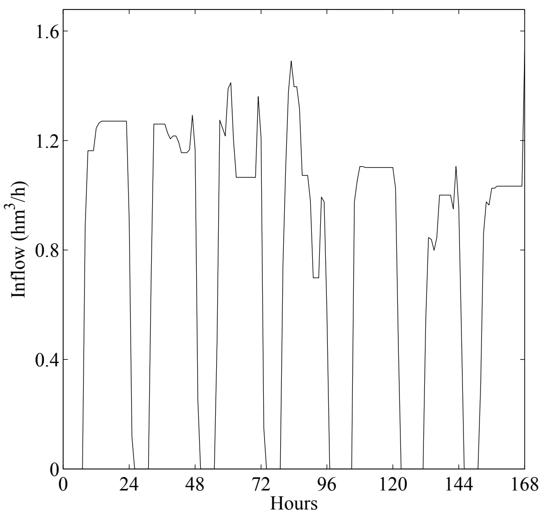

Fig.3 – Inflow on the first reservoir.

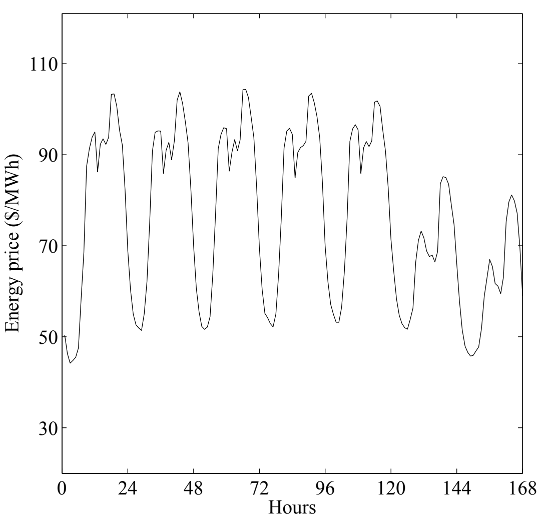

Fig.4 – Energy price profile considered.

Energy prices are important input data to achieve a successful schedule based on profit maximization. This data has uncertainty due to the deregulation of the electricity markets. Hence, an accurate forecast of energy prices has become a very important tool for a GENCO to develop an appropriate bidding strategy in the market and to optimally schedule its hydro resources. Several techniques have been tried out for energy prices forecasting, mainly based on time series and ARIMA models [15], or on artificial neural networks [16]. These energy prices are considered as deterministic input data for the STHS problem.

A comparison of the power generation per water discharge between the LP method and the proposed NLP method for plant 1, 2 and 3 is shown respectively in Fig.5, Fig.6 and Fig.7. In the next figures, the dashed lines denote LP results while the solid lines denote NLP results.

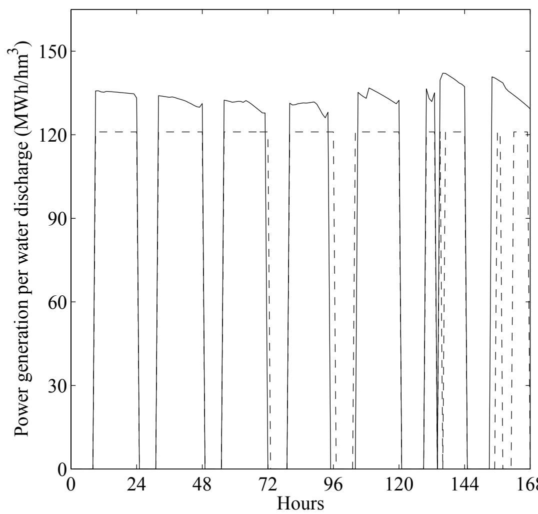

Fig.5 – Power generation per water discharge at plant 1.

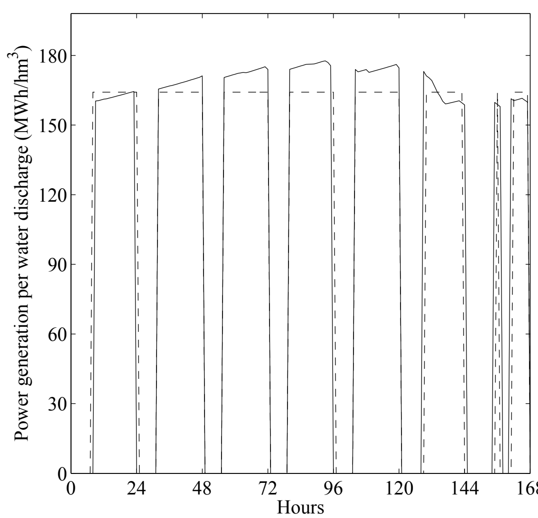

Fig.6 – Power generation per water discharge at plant 2.

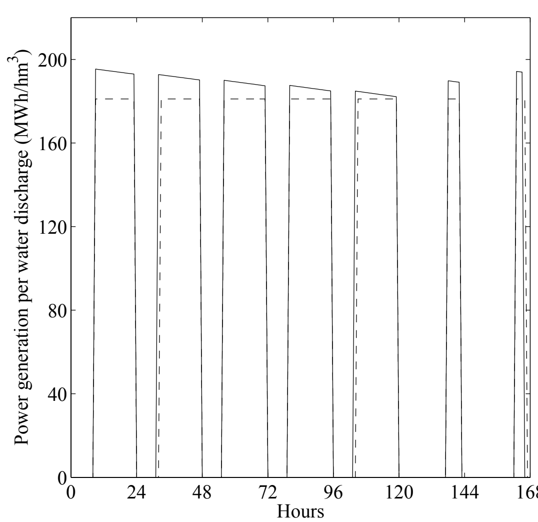

Fig.7 – Power generation per water discharge at plant 3.

This comparison is in favor of the proposed NLP method, achieving a higher total profit with an increase of 4.94% as shown in the Table 1.

Method	Total Profit (k$)	% Increase	CPU (s)
LP	5259.872	-	1.15
NLP	5519.738	4.94	18.79

Table 1 – Comparison of LP with the NLP method.

The computation time for this case study was about 18.79 s, showing that the proposed NLP method is not only more accurate but also computationally acceptable.

6. Conclusion

This paper proposes a nonlinear programming method for head-sensitive cascaded reservoirs in order to consider the head change effect on hydroelectric power generation. As a new contribution to earlier studies, we report the consideration of a slope shape for power generation at the most favorable price hours of each day, instead of the normal flat shape when the maximum power generation was considered with no head change effect. The proposed method has been successfully tested on a case study based on one of the Portuguese cascaded hydro systems with head-sensitive reservoirs, providing a higher profit in comparison with classical optimization methods based on linear programming that ignore head dependence.

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