**Proceedings of the Seventh IASTED International Conference**
**POWER AND ENERGY SYSTEMS**
**August 29–31, 2007, Palma de Mallorca, Spain**
**ISBN Hardcopy: 978-0-88986-689-8 / CD: 978-0-88986-690-4**

# COST-EFFECTIVE GRID STATE IDENTIFICATION OF MEAN VOLTAGE GRIDS BY USING SENSITIVITY ANALYSIS

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**ABSTRACT**
Due to decentralized generation power flow situation in low and mean voltage (MV) level grids has changed which now complicates estimation of grid condition. A satisfying identification necessitates installation of measurement technique because for historical reasons measurement devices are sparsely distributed in MV level grids.

In this paper a new approach to quickly find applicable and cost-effective measurement facilities by using sensitivity analysis and integer optimization algorithms is introduced. Therefore grid equations are remodelled to fit measurable values. After that sensitivity indices are calculated to find optimal measurement locations and measurands.
In this paper nodal measurements of current and power are focused.

**KEY WORDS**
Decentralized generation, measurement, grid condition, sensitivity analysis, under-determined state vector, integer optimization

## 1. Introduction

The increasing amount of decentralized generators results in dramatically changed demands on mean voltage level grids originally planned as distribution grids with one way power flow direction [1]. The high variability of load and generation especially feeders based on renewable sources like global irradiance or wind speed results in fast changing, hardly determinable and possibly illegal grid states. Due to missing measurement technique equipment overstressing cannot be detected adequately so considering even more small feeders measurement devices have to be installed as a precaution.

Integer programming algorithms are used to find valuable and cost-effective measurands and locations for grid condition identification. Unfortunately these algorithms are extremely time-consuming and often unable to find an adequate solution within tolerable time. That is why alternative methods are needed and investigation on sensitivity analysis is done.

## 2. Sensitivity Analysis

Sensitivity analysis is used to systematically determine the effect of any given variables on any interesting variables. Linearization at the operation point results in a sensitivity matrix whose coefficients show the impact of any influencing variable on all state variables which are nodal voltages magnitudes and phases in electrical energy supply grids. Measurable influence variables are magnitudes of root mean square (rms) values of current, voltage and apparent power as well as active and reactive power. In the following subsections accordant sensitivity matrices are developed.

### 2.1 Nodal currents

As mentioned above solely magnitudes of rms-values of current are measurable. The complex nodal current equation [2]

$$\underline{\boldsymbol{Y}}_{\mathrm{KK}}\ \underline{\boldsymbol{u}}_{\mathrm{K}} = \underline{\boldsymbol{i}}_{\mathrm{K}} \tag{1}$$

has to be reformed into the nonlinear equations

$$\begin{aligned}\begin{bmatrix}\varphi_{\mathrm{K}1}\\ \vdots \\ \varphi_{\mathrm{K}n}\end{bmatrix} &= \boldsymbol{f}_{\varphi\mathrm{K}}\left(\begin{bmatrix}\delta_{\mathrm{K}1}\\ \vdots \\ \delta_{\mathrm{K}n}\end{bmatrix},\begin{bmatrix}U_{\mathrm{K}1}\\ \vdots \\ U_{\mathrm{K}n}\end{bmatrix}\right)\\ \begin{bmatrix}I_{\mathrm{K}1}\\ \vdots \\ I_{\mathrm{K}n}\end{bmatrix} &= \boldsymbol{f}_{\mathrm{IK}}\left(\begin{bmatrix}\delta_{\mathrm{K}1}\\ \vdots \\ \delta_{\mathrm{K}n}\end{bmatrix},\begin{bmatrix}U_{\mathrm{K}1}\\ \vdots \\ U_{\mathrm{K}n}\end{bmatrix}\right)\end{aligned} \tag{2}$$

which can now be linearized at the operation point

$$\begin{bmatrix}\varphi_{K1}\\ \vdots\\ \varphi_{Kn}\\ \hdashline I_{K1}\\ \vdots\\ I_{Kn}\end{bmatrix}=\begin{bmatrix}\varphi_{K1,0}\\ \vdots\\ \varphi_{Kn,0}\\ \hdashline I_{K1,0}\\ \vdots\\ I_{Kn,0}\end{bmatrix}+\begin{bmatrix}\Delta\varphi_{K1}\\ \vdots\\ \Delta\varphi_{Kn}\\ \hdashline \Delta I_{K1}\\ \vdots\\ \Delta I_{Kn}\end{bmatrix}=\begin{bmatrix}\boldsymbol{f}_{\varphi K}\\ \hdashline \boldsymbol{f}_{IK}\end{bmatrix}_{OP}+\boldsymbol{J}_{I,KK}\begin{bmatrix}\Delta\delta_{K1}\\ \vdots\\ \Delta\delta_{Kn}\\ \hdashline \Delta U_{K1}\\ \vdots\\ \Delta U_{Kn}\end{bmatrix} \quad (3)$$

The Jacobian $\boldsymbol{J}_{I,KK}$ is build as shown in eq. (4).

$$\boldsymbol{J}_{I,KK}=\begin{bmatrix}\frac{\partial\varphi_{K1}}{\partial\delta_{K1}} & \cdots & \frac{\partial\varphi_{K1}}{\partial\delta_{Kn}} & \frac{\partial\varphi_{K1}}{\partial U_{K1}} & \cdots & \frac{\partial\varphi_{K1}}{\partial U_{Kn}}\\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots\\ \frac{\partial\varphi_{Kn}}{\partial\delta_{K1}} & \cdots & \frac{\partial\varphi_{Kn}}{\partial\delta_{Kn}} & \frac{\partial\varphi_{Kn}}{\partial U_{K1}} & \cdots & \frac{\partial\varphi_{Kn}}{\partial U_{Kn}}\\ \hdashline \frac{\partial I_{K1}}{\partial\delta_{K1}} & \cdots & \frac{\partial I_{K1}}{\partial\delta_{Kn}} & \frac{\partial I_{K1}}{\partial U_{K1}} & \cdots & \frac{\partial I_{K1}}{\partial U_{Kn}}\\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots\\ \frac{\partial I_{Kn}}{\partial\delta_{K1}} & \cdots & \frac{\partial I_{Kn}}{\partial\delta_{Kn}} & \frac{\partial I_{Kn}}{\partial U_{K1}} & \cdots & \frac{\partial I_{Kn}}{\partial U_{Kn}}\end{bmatrix} \quad (4)$$

The inverse of slack reduced $\boldsymbol{J}_{I,KK}$ consists of four sensitivity matrices.

$$\boldsymbol{J}_{I,KK,red}^{-1}=\begin{bmatrix}\boldsymbol{A}_{\delta K\varphi K} & \boldsymbol{A}_{\delta KIK}\\ \boldsymbol{A}_{UK\varphi K} & \boldsymbol{A}_{UKIK}\end{bmatrix} \quad (5)$$

such that

$$\begin{bmatrix}\Delta\delta_{K1}\\ \vdots\\ \Delta\delta_{Kn}\end{bmatrix}=\boldsymbol{A}_{\delta K\varphi K}\begin{bmatrix}\Delta\varphi_{K1}\\ \vdots\\ \Delta\varphi_{Kn}\end{bmatrix}+\boldsymbol{A}_{\delta KIK}\begin{bmatrix}\Delta I_{K1}\\ \vdots\\ \Delta I_{Kn}\end{bmatrix} \quad (6)$$

and

$$\begin{bmatrix}\Delta U_{K1}\\ \vdots\\ \Delta U_{Kn}\end{bmatrix}=\boldsymbol{A}_{UK\varphi K}\begin{bmatrix}\Delta\varphi_{K1}\\ \vdots\\ \Delta\varphi_{Kn}\end{bmatrix}+\boldsymbol{A}_{UKIK}\begin{bmatrix}\Delta I_{K1}\\ \vdots\\ \Delta I_{Kn}\end{bmatrix} \quad (7)$$

Due to missing measured values of phase angles only $\boldsymbol{A}_{\delta KIK}$ and $\boldsymbol{A}_{UKIK}$ are needed. Eq. (6) and (7) show that grid condition identification based on nodal currents is faulty even if all nodes are equipped with measurement devices.

### 2.2 Nodal active and reactive power

Complementary to nodal currents one can assume that active and reactive power are measurable. The nonlinear complex nodal power equation [2]

$$\underline{\boldsymbol{s}}_K = 3\ \mathrm{diag}\left(\underline{\boldsymbol{u}}_K\right)\left(\underline{\boldsymbol{Y}}_{KK}\ \underline{\boldsymbol{u}}_K\right)^* \quad (8)$$

is divided into real and imaginary part

$$\begin{bmatrix}P_{K1}\\ \vdots\\ P_{Kn}\\ \hdashline Q_{K1}\\ \vdots\\ Q_{Kn}\end{bmatrix}=3\begin{bmatrix}\mathrm{Re}\left\{\mathrm{diag}\left(\underline{\boldsymbol{u}}_K\right)\left(\underline{\boldsymbol{Y}}_{KK}\ \underline{\boldsymbol{u}}_K\right)^*\right\}\\ \hdashline \mathrm{Im}\left\{\mathrm{diag}\left(\underline{\boldsymbol{u}}_K\right)\left(\underline{\boldsymbol{Y}}_{KK}\ \underline{\boldsymbol{u}}_K\right)^*\right\}\end{bmatrix}=$$

$$=\begin{bmatrix}\boldsymbol{f}_{PK}\left(\begin{bmatrix}\delta_{K1}\\ \vdots\\ \delta_{Kn}\end{bmatrix},\begin{bmatrix}U_{K1}\\ \vdots\\ U_{Kn}\end{bmatrix}\right)\\ \hdashline \boldsymbol{f}_{QK}\left(\begin{bmatrix}\delta_{K1}\\ \vdots\\ \delta_{Kn}\end{bmatrix},\begin{bmatrix}U_{K1}\\ \vdots\\ U_{Kn}\end{bmatrix}\right)\end{bmatrix} \quad (9)$$

and linearized at the operation point

$$\begin{bmatrix}P_{K1}\\ \vdots\\ P_{Kn}\\ \hdashline Q_{K1}\\ \vdots\\ Q_{Kn}\end{bmatrix}=\begin{bmatrix}P_{K1,0}\\ \vdots\\ P_{Kn,0}\\ \hdashline Q_{K1,0}\\ \vdots\\ Q_{Kn,0}\end{bmatrix}+\begin{bmatrix}\Delta P_{K1}\\ \vdots\\ \Delta P_{Kn}\\ \hdashline \Delta Q_{K1}\\ \vdots\\ \Delta Q_{Kn}\end{bmatrix}=\begin{bmatrix}\boldsymbol{f}_{PK}\\ \hdashline \boldsymbol{f}_{QK}\end{bmatrix}_{OP}+\boldsymbol{J}_{S,KK}\begin{bmatrix}\Delta\delta_{K1}\\ \vdots\\ \Delta\delta_{Kn}\\ \hdashline \Delta U_{K1}\\ \vdots\\ \Delta U_{Kn}\end{bmatrix} \quad (10)$$

with

$$\boldsymbol{J}_{S,KK}=\begin{bmatrix}\frac{\partial P_{K1}}{\partial\delta_{K1}} & \cdots & \frac{\partial P_{K1}}{\partial\delta_{Kn}} & \frac{\partial P_{K1}}{\partial U_{K1}} & \cdots & \frac{\partial P_{K1}}{\partial U_{Kn}}\\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots\\ \frac{\partial P_{Kn}}{\partial\delta_{K1}} & \cdots & \frac{\partial P_{Kn}}{\partial\delta_{Kn}} & \frac{\partial P_{Kn}}{\partial U_{K1}} & \cdots & \frac{\partial P_{Kn}}{\partial U_{Kn}}\\ \hdashline \frac{\partial Q_{K1}}{\partial\delta_{K1}} & \cdots & \frac{\partial Q_{K1}}{\partial\delta_{Kn}} & \frac{\partial Q_{K1}}{\partial U_{K1}} & \cdots & \frac{\partial Q_{K1}}{\partial U_{Kn}}\\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots\\ \frac{\partial Q_{Kn}}{\partial\delta_{K1}} & \cdots & \frac{\partial Q_{Kn}}{\partial\delta_{Kn}} & \frac{\partial Q_{Kn}}{\partial U_{K1}} & \cdots & \frac{\partial Q_{Kn}}{\partial U_{Kn}}\end{bmatrix} \quad (11)$$

The inverse of the slack reduced jacobian

$$\boldsymbol{J}_{S,KK,red}^{-1}=\begin{bmatrix}\boldsymbol{A}_{\delta KPK} & \boldsymbol{A}_{\delta KQK}\\ \boldsymbol{A}_{UKPK} & \boldsymbol{A}_{UKQK}\end{bmatrix} \quad (12)$$

consists as well of four sensitivity matrices such that

$$\begin{bmatrix}\Delta\delta_{K1}\\ \vdots\\ \Delta\delta_{Kn}\end{bmatrix}=\boldsymbol{A}_{\delta KPK}\begin{bmatrix}\Delta P_{K1}\\ \vdots\\ \Delta P_{Kn}\end{bmatrix}+\boldsymbol{A}_{\delta KQK}\begin{bmatrix}\Delta Q_{K1}\\ \vdots\\ \Delta Q_{Kn}\end{bmatrix} \quad (13)$$

and

$$\begin{bmatrix}\Delta U_{K1}\\ \vdots\\ \Delta U_{Kn}\end{bmatrix}=\boldsymbol{A}_{UKPK}\begin{bmatrix}\Delta P_{K1}\\ \vdots\\ \Delta P_{Kn}\end{bmatrix}+\boldsymbol{A}_{UKQK}\begin{bmatrix}\Delta Q_{K1}\\ \vdots\\ \Delta Q_{Kn}\end{bmatrix} \quad (14)$$

Eq. (13) and (14) show that measuring active and reactive power at all nodes is accurate but as well very expensive.

### 2.3 Nodal apparent power

Nodal apparent power can easily be calculated if nodal voltage and current is known or measured. Due to missing information on phase angles calculation of apparent power sensitivity matrices is analogue to nodal current sensitivity matrix calculation.
Eq. (8) is reformed into

$$\begin{bmatrix}\varphi_{K1}\\ \vdots \\ \varphi_{Kn}\end{bmatrix} = \boldsymbol{f}_{\varphi K}\left(\begin{bmatrix}\delta_{K1}\\ \vdots \\ \delta_{Kn}\end{bmatrix},\begin{bmatrix}U_{K1}\\ \vdots \\ U_{Kn}\end{bmatrix}\right)$$
$$\begin{bmatrix}S_{K1}\\ \vdots \\ S_{Kn}\end{bmatrix} = \boldsymbol{f}_{SK}\left(\begin{bmatrix}\delta_{K1}\\ \vdots \\ \delta_{Kn}\end{bmatrix},\begin{bmatrix}U_{K1}\\ \vdots \\ U_{Kn}\end{bmatrix}\right) \tag{15}$$

and linearized at the operation point such that

$$\boldsymbol{J}_{S,KK} = \left[\begin{array}{ccc|ccc}\frac{\partial \varphi_{K1}}{\partial \delta_{K1}} & \cdots & \frac{\partial \varphi_{K1}}{\partial \delta_{Kn}} & \frac{\partial \varphi_{K1}}{\partial U_{K1}} & \cdots & \frac{\partial \varphi_{K1}}{\partial U_{Kn}} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial \varphi_{Kn}}{\partial \delta_{K1}} & \cdots & \frac{\partial \varphi_{Kn}}{\partial \delta_{Kn}} & \frac{\partial \varphi_{Kn}}{\partial U_{K1}} & \cdots & \frac{\partial \varphi_{Kn}}{\partial U_{Kn}} \\ \hline \frac{\partial S_{K1}}{\partial \delta_{K1}} & \cdots & \frac{\partial S_{K1}}{\partial \delta_{Kn}} & \frac{\partial S_{K1}}{\partial U_{K1}} & \cdots & \frac{\partial S_{K1}}{\partial U_{Kn}} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial S_{Kn}}{\partial \delta_{K1}} & \cdots & \frac{\partial S_{Kn}}{\partial \delta_{Kn}} & \frac{\partial S_{Kn}}{\partial U_{K1}} & \cdots & \frac{\partial S_{Kn}}{\partial U_{Kn}}\end{array}\right] \tag{16}$$

The slack reduced inverse of $\boldsymbol{J}_{S,KK}$ consists again of four sensitivity matrices

$$\boldsymbol{J}_{S,KK,red}^{-1} = \left[\begin{array}{c|c}\boldsymbol{A}_{\delta K\varphi K} & \boldsymbol{A}_{\delta KSK} \\ \hline \boldsymbol{A}_{UK\varphi K} & \boldsymbol{A}_{UKSK}\end{array}\right] \tag{17}$$

such that

$$\begin{bmatrix}\Delta\delta_{K1}\\ \vdots \\ \Delta\delta_{Kn}\end{bmatrix} = \boldsymbol{A}_{\delta K\varphi K}\begin{bmatrix}\Delta\varphi_{K1}\\ \vdots \\ \Delta\varphi_{Kn}\end{bmatrix} + \boldsymbol{A}_{\delta KSK}\begin{bmatrix}\Delta S_{K1}\\ \vdots \\ \Delta S_{Kn}\end{bmatrix} \tag{18}$$

and

$$\begin{bmatrix}\Delta U_{K1}\\ \vdots \\ \Delta U_{Kn}\end{bmatrix} = \boldsymbol{A}_{UK\varphi K}\begin{bmatrix}\Delta\varphi_{K1}\\ \vdots \\ \Delta\varphi_{Kn}\end{bmatrix} + \boldsymbol{A}_{UKSK}\begin{bmatrix}\Delta S_{K1}\\ \vdots \\ \Delta S_{Kn}\end{bmatrix} \tag{19}$$

Due to missing phase angle information only $\boldsymbol{A}_{\delta KSK}$ and $\boldsymbol{A}_{UKSK}$ are of interest. That is why apparent power measurement is faulty as well even if all nodes are equipped with measurement facilities analogue to nodal currents.

## 3. Interpretation

### 3.1 Sensitivity Index

The coefficient $a_{ij}$ of a sensitivity matrix $\boldsymbol{A}$ states the effect of influence variable $j$ on state variable $i$. Its magnitude shows the strength of influence while its sign expresses the direction of alteration.

To evaluate the quality of each measurand its sensitivity index is calculated by summing magnitudes of coefficients of each corresponding column. A higher sensitivity index denotes a better measurand. Secondary the index shows operation point dependency as well.

### 3.2 Error estimation

Assuming at most one measuring device per location the actual error can be calculated with the conglomerated sensitivity matrix $\boldsymbol{A}$, the binary device vector $\boldsymbol{x}$, the measured or forecasted influence vector $\boldsymbol{m}$ and the deviation of nodal voltages.

$$\boldsymbol{f}_{act} = \boldsymbol{A}\begin{bmatrix}x_1 & & \\ & \ddots & \\ & & x_n\end{bmatrix}\boldsymbol{m} - \left[\begin{array}{c}\delta_{1,0} - \delta_{1,AP}\\ \vdots \\ \delta_{n,0} - \delta_{n,AP} \\ \hline U_{1,0} - U_{1,AP} \\ \vdots \\ U_{n,0} - U_{n,AP}\end{array}\right] \tag{20}$$

Since the actual deviation is unknown it is best approximated by

$$\left[\begin{array}{c}\delta_{1,0} - \delta_{1,AP}\\ \vdots \\ \delta_{n,0} - \delta_{n,AP} \\ \hline U_{1,0} - U_{1,AP} \\ \vdots \\ U_{n,0} - U_{n,AP}\end{array}\right] \cong \boldsymbol{A}\begin{bmatrix}1 & & \\ & \ddots & \\ & & 1\end{bmatrix}\boldsymbol{m} \tag{21}$$

so

$$\boldsymbol{f}_{est} = \boldsymbol{A}\left(\begin{bmatrix}x_1 & & \\ & \ddots & \\ & & x_m\end{bmatrix} - \boldsymbol{E}\right)\boldsymbol{m} \tag{22}$$

with

$$\boldsymbol{f}_{est} \geq \boldsymbol{f}_{act} \tag{23}$$

### 3.3 Optimization

The optimization problem can be formulated as follows [3], [4]:

$$\min\left(\boldsymbol{c}^{\mathrm{T}} \cdot \boldsymbol{x}\right)$$

$$\text{s. t.: } \boldsymbol{A}\begin{bmatrix} m_1 & & \\ & \ddots & \\ & & m_n \end{bmatrix} \boldsymbol{x} \leq \boldsymbol{f}_{\max} + \boldsymbol{A}\,\boldsymbol{m} \qquad (24)$$

with the objective function $\boldsymbol{c}$ containing the investment cost of each measurement device and the tolerable error $\boldsymbol{f}_{\max}$. Optimization algorithms can be looked up in references and will not be expatiated in this paper.

Assuming a single loop grid of 50 nodes the algorithm has to handle $3.8 \cdot 10^{30}$ possible solutions considering nodes, terminals and the mentioned different measurands. Even this simple problem is impossible to be solved in a tolerable amount of time. To decrease optimization time a good starting vector is necessary. It can be obtained by adding devices, sorted by its weighted sensitivity index, to the binary vector $\boldsymbol{x}$ until the constraint is satisfied. The resulting $\boldsymbol{x}$ now needs only few modifications depending on the objective function which can be calculated quickly.

## 4. Simple example

Given are the demonstration grid in Fig.1 and its parameters in Table1. The operator wants to cost-effectively identify grid condition by installing either current or apparent power rms indicators.

Furthermore it is assumed that investment cost per ammeter is 1 unit, investment cost per power meter is 1.5 units and the operator tolerates a maximum error of 0.2 % which is about 23 V in 20-kV-grids.

Due to entirely missing measurement equipment grid condition is completely unknown except the slack voltage at node 9 which can be assumed as 20 kV. The given nodal powers in Fig.1 can be obtained from substation transformer labels and are the highest possible values.

**Table 1** Parameters of all lines

| | |
|---|---|
| length | 1 km |
| *r'* | 0.2 Ohm/km |
| *x'* | 0.15 Ohm/km |
| *c'* | 300 nF/km |

As mentioned above the actual nodal powers and currents are unknown. Investigation is done to examine which powers or accordant currents have to be measured. Thus sensitivity matrices are calculated. The indices are given in Table 2.

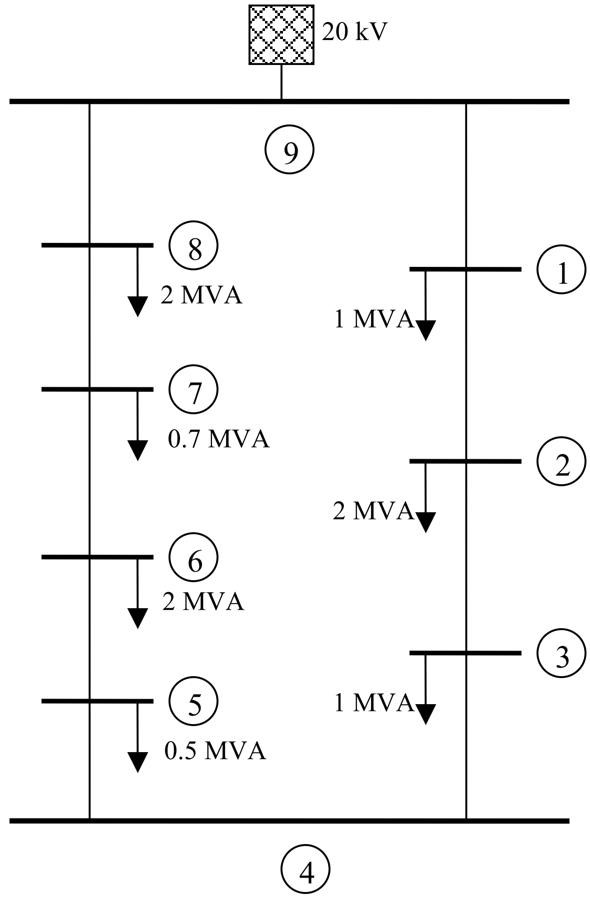


**Fig. 1** demonstration grid

**Table 2** Sensitivity indices

| node | Current SI | Power SI | Weighted Current SI | Weighted Power SI |
|---|---|---|---|---|
| 1 | 0.7994 | 0.7991 | 40,1 | 23,1 |
| 2 | 1.4002 | 1.4004 | 140,6 | 80,9 |
| 3 | 1.8009 | 1.8012 | 90,4 | 52 |
| 4 | 0 | 0 | 0 | 0 |
| 5 | 2.0013 | 2.0019 | 50,3 | 28,9 |
| 6 | 1.8011 | 1.802 | 181 | 104 |
| 7 | 1.4001 | 1.4002 | 49,2 | 28,3 |
| 8 | 0.7995 | 0.7992 | 80,1 | 46,1 |
| 9 | 0 | 0 | 0 | 0 |

As to be seen in Table2 measurement of current at node 6 is added to $\boldsymbol{x}$. However this starting vector does not satisfy the constraints so according to Table2 measurement of current at node 2 is added. The new vector leads to a maximum error of 9.4 V and thus is used as starting vector.

In this case the optimal measurement vector is already found because power meters are more expensive than ammeters. Sensitivity analysis avoided optimum search within 1024 solution.

To show operation point dependency a set of 60 different load scenarios was generated containing all operation points varying from idling to full load. Fig.2 shows operation point dependency of nodal currents and Fig.3 of nodal apparent power. One can see that changes in sensitivity only take place when nodal values change sign or are set to zero.

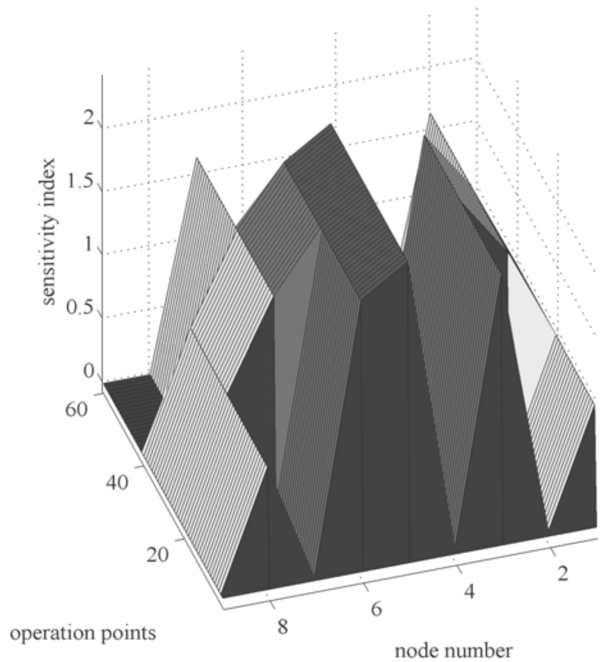


**Fig. 2** operation point dependency of current sensitivity index

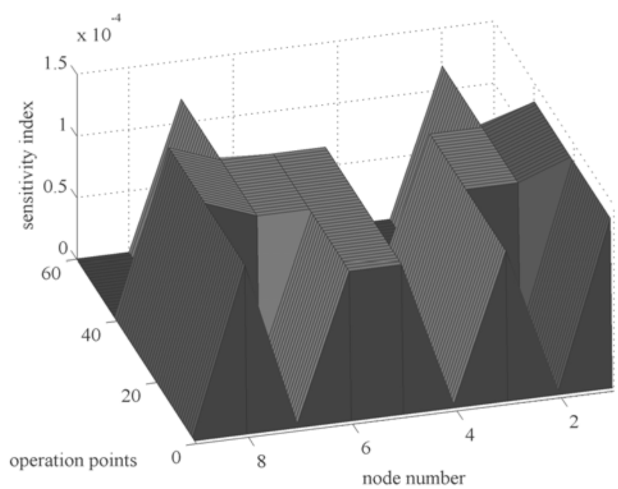


**Fig. 3** operation point dependency of power sensitivity index

## 5. Conclusion

Sensitivity analysis is a fast and systematic way to find applicable and cost-effective measurands and measurement locations for grid condition identification. All measuring possibilities are rated and can be chosen by varying constraints. The need of integer optimization and therewith calculation time is reduced to a minimum.

## References

[1] M. Wolter, Power flow simulation of autonomous MicroGrids fed by renewable sources, *WSEAS Transactions on Power Systems, 1*(11), 2006, 1911-1917.

[2] J. J. Grainger, W. D. Stevenson, *Power system analysis* (New York, NY: McGraw-Hill, 1994).

[3] D. Li, X. Sun, *Nonlinear integer programming* (New York, NY: Springer, 2006).

[4] J. K. Karlof, *Integer programming: theory and practice* (Boca Raton, Fla: Taylor & Francis, 2006)

[5] A. Abur, A. G. Expósito, *Power system state estimation: theory and implementation* (New York, NY: Dekker, 2004).

[6] P. S. Kundur, N. J. Balu, *Power system stability and control* (New York, NY: McGraw-Hill, 1994)

[7] S. Conti, A. M. Greco, S. Raiti, A simplified approach to voltage sensitivity analysis in radial LV and MV distribution networks, *WSEAS Transactions on Power Systems, 1*(11), 2006, 1837-1843.

[8] B. N. Pshenichnyj, *The linearization method for constrained optimization* (Berlin, D: Springer, 1994).