Proceedings of the Seventh IASTED International Conference
POWER AND ENERGY SYSTEMS
August 29–31, 2007, Palma de Mallorca, Spain
ISBN Hardcopy: 978-0-88986-689-8 / CD: 978-0-88986-690-4

STATIC VOLTAGE STABILITY ANALYSIS OF POWER SYSTEMS CONSIDERING INDUCTION MOTOR COMPONENTS OF LOADS

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ABSTRACT
For static voltage stability studies of a power system, the loading of the system is increased incrementally and slowly (in certain direction) to the point of voltage collapse. The MW-distance to this point is a good measure of system voltage stability limit. The voltage profile of the system is shown by the PV-curves which are plotted using continuation power flow programs as the loading varies from the base values to the point of collapse. Most reported studies consider constant PQ (or at best constant impedance, constant current, constant power, called ZIP) models for loads. Given, in average, 60% of the loads include induction motors which do not follow the ZIP-model, the results obtained from these PV-curves may be erroneous. In this paper, voltage stability analysis is performed through continuation power flow program that accommodates the model for the induction motor components of the loads. The method is tested using the New England 39-bus Power System. Comparison of the presented method with the commonly used method shows that the voltage stability limits found commonly can be too optimistic. It was found that when induction motor components of the loads are considered, the obtained voltage stability limit is smaller and the overall voltage profile of the system is very much different than that found using only PQ-load models.

KEY WORDS
Voltage collapse, voltage stability, continuation power flow, and induction motor model

1. Introduction

Voltage stability is an important aspect of power system planning, operation, and control. The ability to maintain voltage stability for today's stressed power systems is a growing concern and requires careful voltage stability studies. Accurate voltage stability analysis of a power system involves digital simulation using dynamic and algebraic equations that represents the comprehensive model of the system. However, such simulations requires huge amount of computer time which makes it impractical to be used for real time monitoring of system voltage.

Static voltage stability analysis provides an estimate of the overall power system voltage stability. If properly used, this analysis can provide much insight into the voltage instability/collapse problem. A common measure of static voltage stability limit (or voltage stability margin) is the MW-distance to the point of collapse. Eigenvalue analysis, singular value decomposition, voltage stability indices, QV and PV curves are the commonly used methods for static voltage stability studies [1:9]. Only the PV-curves, obtained by continuation power flow program that clearly shows the MW-distance to the point of collapse and the system voltage profile is used in this paper.

Commonly, constant PQ, or ZIP (which stands for constant impedance, constant current, constant power) load models are used in continuation power flow program for obtaining the system PV-curves. Over 60% of the electric power is consumed by induction machines. The models for these machines are neither constant PQ nor ZIP. The reactive power consumption of an induction machine is very much related to its active power consumption (or equivalently its slip) [10:16]. Consequently the results obtained from common static voltage stability analysis could be erroneous.

The objective of this paper is to accommodate the induction machine components of the loads into the static voltage stability analysis. This is done by using an equivalent induction machine model, with aggregate parameters [17, 18], for each bus to accommodate for the induction motor component of the load at that bus.

The proposed method is implemented in MATLAB and tested using the New England 39-bus power system. Numerical results show that the voltage stability limit and the voltage profile of the system found with the proposed method are significantly different than the ones found with the common method. Given the presented method considers load model that is closer to real physical loads, it can be concluded that the commonly used method for static voltage stability may give erroneous results.

The organization of this paper is as follows. Section 2 gives a review and a different presentation for power flow model including induction machines. The proposed method of including induction motor load component into voltage stability analysis process is presented in Section 3. This method is applied to a sample power system in Section 4 and Section 5 concludes the paper.

2. Power Flow Including Induction Motors

2.1 Steady State Model of Induction Motor

In steady state operation, an induction motor may be presented by the standard equivalent circuit as shown in Fig. 1 [10], where R_s / X_s are the stator resistance / leakage reactance, R_r / X_r are the rotor resistance / leakage reactance referred to stator side, X_m is the magnetizing reactance referred to stator side, and s is the rotor slip. The circuit model is given in p.u. Here, V represents the magnitude of the voltage of the bus to which the induction motor is connected.

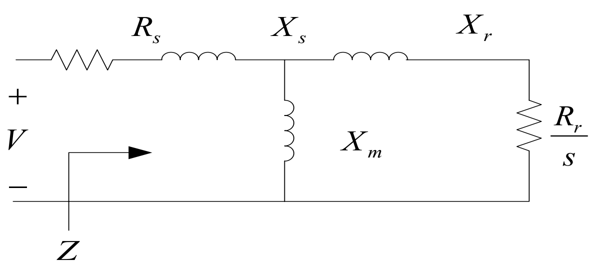

Figure 1: Equivalent Circuit Model for Steady State Operation

The equivalent impedance of the motor (Z) as seen by the power system at this bus is given by:

$$Z = R_s + jX_s + (jX_m) \| \left(\frac{R_r}{s} + jX_r \right) \quad (1)$$

The complex power S received by the motor is calculated using:

$$S = V^2 / Z^* \quad (2)$$

After much simplification and separating the real and imaginary parts of S, and assuming :

$$X = X_s X_m + X_s X_r + X_m X_r$$
$$X_1 = X_s + X_m$$
$$X_2 = X_m + X_r$$

the active and reactive power received by the motor are found to be:

$$P = \frac{R_s R_r^2 - R_r s X + R_r s X_2 X_1 + R_s s^2 X_2^2}{(R_s R_r - sX)^2 + (R_r X_1 + R_s s X_2)^2} V^2 \quad (3)$$

$$Q = \frac{R_r^2 X_1 + R_r R_s s X_2 - R_s R_r X_2 s + s^2 X_2 X}{(R_s R_r - sX)^2 + (R_r X_1 + R_s s X_2)^2} V^2 \quad (4)$$

The above equations involve four variables: active power P, reactive power Q, bus voltage magnitude V, and slip s. For known values of P and V, (3) represents a second order algebraic equation in terms of slip s as follows:

$$a s^2 + b s + c = 0 \quad (5)$$

where a, b, and c can be derived from (3) which gives the following results:

$$a = PX^2 + PR_s^2 X_2^2 - R_s X_2^2 V^2 \quad (6)$$
$$b = 2PR_s R_r (X_1 X_2 - X) - R_r V^2 (X_1 X_2 - X) \quad (7)$$
$$c = R_r^2 (PR_s^2 + PX_1^2 - R_s V^2) \quad (8)$$

Now, the induction motor reactive power can be calculated by plugging V and s in (4).

2.2 Power Flow Equations with Induction Motor

Figure 2 shows a generic load bus (Bus *i*) of a power system that includes both constant PQ-load and induction motor load. In this figure, $P_{M,i}$ ($Q_{M,i}$) represents the active (reactive) power received by the motor, $P_{L,i}$ ($Q_{L,i}$) represents the active (reactive) power received by the constant PQ-load, and P_i (Q_i) represents the net active (reactive) power injected to the power system network at this bus.

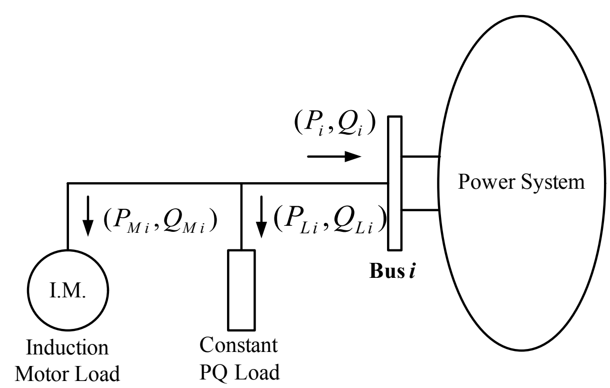

Figure 2: Load Representation at a Generic Bus *i*

From this figure, the power flow equations for Bus *i* are given as follows:

$$P_i^{sch} = -P_{M,i} - P_{L,i} = \sum_{j=1}^{N} V_i V_j Y_{ij} \cos(\delta_i - \delta_j - \theta_{ij}) \quad (9)$$

$$Q_i^{sch} = -Q_{L,i} = Q_{M,i} + \sum_{j=1}^{N} V_i V_j Y_{ij} \sin(\delta_i - \delta_j - \theta_{ij}) \quad (10)$$

Where V_i (δ_i) is the magnitude (angle) of the voltage at Bus *i*, Y_{ij} (θ_{ij}) is the magnitude (angle) of the *ij*-entry of the system Y-matrix, and N is the total number of buses.

The active power $P_{M,i}$ and $P_{L,i}$ are known values. Also, while $Q_{L,i}$ is a known value, $Q_{M,i}$ need be calculated through the power flow solution. P_i^{sch} and Q_i^{sch} are defined as the scheduled active and reactive power at Bus *i* which have to be satisfied through the power flow solution.

2.3 Modification of the Newton-Raphson Method

Slight modifications are needed to the existing Newton-Raphson method of power flow solution to accommodate loads involving induction motors. Comparing the above equations with those of the case with only PQ-load, we conclude:

1) The scheduled net injected active power is now ($-P_{M,i} - P_{L,i}$) instead of $-P_{L,i}$,
2) The scheduled net injected reactive power is ($-Q_{L,i}$),
3) All entries of the Jacobian Matrix related to the active power are unchanged, and
4) All entries of the Jacobian Matrix related to reactive power are unchanged except for the diagonal element $\partial Q_i / \partial V_i$ which would have an added term of $\partial Q_{M,i} / \partial V_i$. This added term is calculated using (4). After simplification this term is given by:

$$\frac{\partial Q_{M,i}}{\partial V_i} = \frac{2Q_{M,i}}{V_i} \quad (11)$$

The iterative Newton-Raphson solution is carried out as usual except that, in each iteration: (5) has to be solved for slip s, $Q_{M,i}$ has to be calculated from (4), and $\partial Q_{M,i} / \partial V_i$ has to be calculated from (11) and then added to the diagonal elements $\partial Q_i / \partial V_i$ for buses that include induction motor loads.

3. Voltage Study Including Induction Motors

PV-Curve is one of the most important tools for static voltage stability study. These curves are based on continuation power flow and show how the voltage profile of a power system varies as the loading is increased from a base value to the largest possible maximum value (called the MW-distance to the point of collapse) in certain prescribed direction. The continuation power flow is based on steady state power flow analysis which is typically formulated (assuming constant PQ-loads) as follows:

$$P_i^{sch} = P_{io} + d_i \lambda P_{io} = P_{io}(1 + d_i \lambda) = \sum_{j=1}^{N} V_i V_j Y_{ij} \cos(\delta_i - \delta_j - \theta_{ij}) \quad (12)$$

$$Q_i^{sch} = Q_{io} + \hat{d}_i \lambda Q_{io} = Q_{io}(1 + \hat{d}_i \lambda) = \sum_{j=1}^{N} V_i V_j Y_{ij} \sin(\delta_i - \delta_j - \theta_{ij}) \quad (13)$$

Where:

P_{io} (Q_{io}) is the base value of the net injected active (reactive) power at Bus *i*,

d_i ($\hat{d}_i$) is the relative or proportionate increase in active (reactive) power at Bus *i*,

λ is the changing parameter whose value is zero at base loading, and

Other variables were defined previously.

The voltage stability limit is determined by finding the maximum (or critical) value of λ (called λ_c) that satisfies the above set of equations. The voltage profile of the system is plotted as λ varies between 0 (base loading) and λ_c (maximum loading).

If the load includes induction motor component, the above equations would have to be modified as follow:

$$P_i^{sch} = -P_{M,io}(1 + d_{M,i}\lambda) - P_{L,io}(1 + d_{L,i}\lambda) = \sum_{j=1}^{N} V_i V_j Y_{ij} \cos(\delta_i - \delta_j - \theta_{ij}) \quad (14)$$

$$Q_i^{sch} = -Q_{L,io}(1 + \hat{d}_{L,i}\lambda) = +Q_{M,i} + \sum_{j=1}^{N} V_i V_j Y_{ij} \sin(\delta_i - \delta_j - \theta_{ij}) \quad (15)$$

Where $P_{M,io}$, $P_{L,io}$, and $Q_{L,io}$ are the base values of $P_{M,i}$, $P_{L,i}$, and $Q_{L,i}$ respectively.

According to these equations, the active power of the induction motor and the PQ-load, as well as the reactive power of the PQ-load are allowed to increase in any prescribed directions (independent of one another). However, the reactive power of the induction motor load cannot be selected arbitrarily, and has to be calculated from (4) which depends on the selection of the induction motor active power (through the slip). As shown through the numerical analysis in next section, this has profound effects on the voltage stability limit and the voltage profile of the system.

4. Numerical Results

The New England 39-Bus System is used to show the effects of the induction motor component of loads on static voltage stability limit and the PV-curves. The induction motor components of the loads in the original system are not given. These loads were assumed 100% constant PQ-loads by other authors for similar studies. In this paper, certain percentages of the loads at different buses were assumed to be induction motor loads. These percentages were chosen arbitrarily, but carefully, to make sure the overall percentage for the induction motor component of the overall load is around 60% which is typical of any power system.

Table 1 shows the aggregate induction machine parameters. The last column indicates the percentage of the loads that are consumed by induction machines. The average value for the entries of this column is 60 %. Note that induction machines are considered only for the buses that have existing loads. Figures 4 and 5 show the PV-curves for the associated load buses for the two cases of 100% PQ-loads and combined induction motor and PQ-loads, respectively. The PQ-loads were assumed to be constant power factor.

The PV-curves in Figures 4 and 5 reveals that the results from voltage stability analysis performed assuming 100% PQ-loads is very optimistic both in terms of MW-distance to voltage collapse and the voltage profile of the system. With pure PQ-loads we obtain the critical loading parameter of λ_c =1.0924 while including the induction motor components of the loads gives λ_c =0.9363. In addition, the voltage profile of the system for any loading factor of λ is lower when induction motor load components are included in the analysis. This numerical results should be expected due to the well known fact that induction motors consume a large amount of reactive power when the active power consumption increases and/or the terminal voltage decreases.

5. Conclusion

This paper presents a method to include the induction motor components of the loads for static voltage stability analysis by integrating the steady state model of induction machines into the continuation power flow program. The loading of the New England 39-bus System was modified to include induction motor component and then was used as the test system in this paper. Comparison of the results obtained using the presented method with those of the commonly used method reveals that the voltage stability limits found commonly can be too optimistic both in terms MW-distance to voltage collapse and the voltage profile of the system.

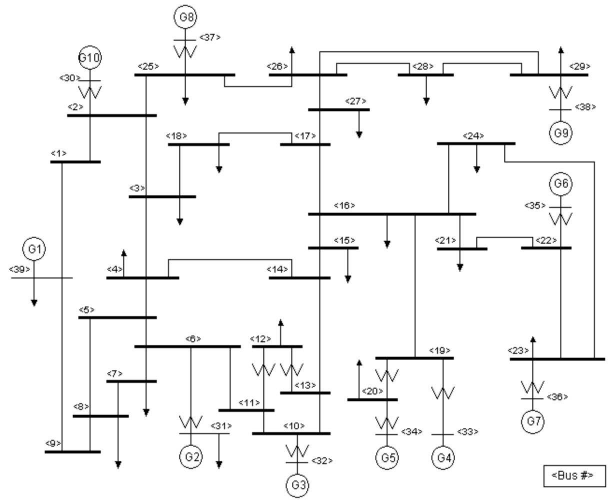

Figure 3: The New England 39-Bus System.

Table 1: Aggregate Parameters of Induction Motors at different Buses

Bus	R_s	X_s	X_m	X_r	R_r	% of Load
3	0.0123	0.0508	0.857	0.0508	0.0825	90
4	0.0687	0.1507	2.57	0.1237	0.037	44
7	0.064	0.091	2.23	0.071	0.059	28
8	0.077	0.107	2.22	0.098	0.079	62
12	0.0123	0.0508	0.857	0.0508	0.0825	62
15	0.0172	0.1058	3.05	0.1028	0.0228	33
16	0.077	0.107	2.22	0.098	0.079	60
18	0.064	0.091	2.23	0.071	0.059	76
20	0.0172	0.1058	3.05	0.1028	0.0228	79
21	0.064	0.091	2.23	0.071	0.059	66
23	0.035	0.094	2.8	0.163	0.048	43
24	0.035	0.094	2.8	0.163	0.048	59
25	0.0687	0.1507	2.57	0.1237	0.037	68
26	0.0175	0.0703	1.05	0.0703	0.109	71
27	0.0175	0.0703	1.05	0.0703	0.109	70
28	0.0687	0.1507	2.57	0.1237	0.037	50
29	0.064	0.091	2.23	0.071	0.059	57

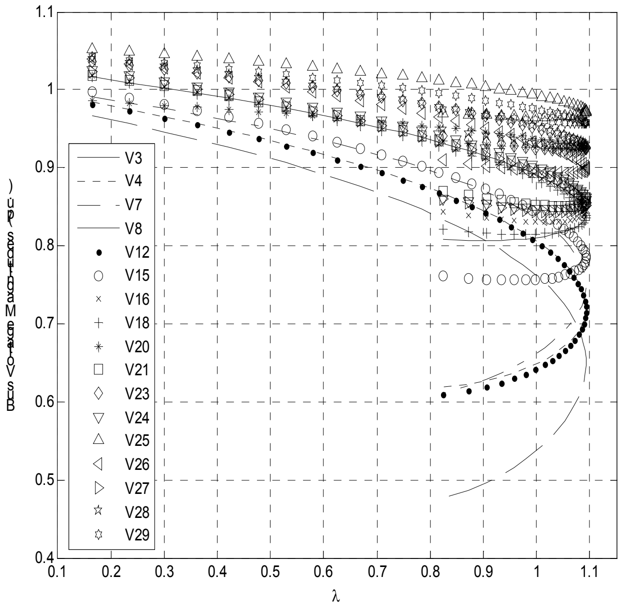

Figure 4: PV-Curve Assuming 100% PQ-Loads

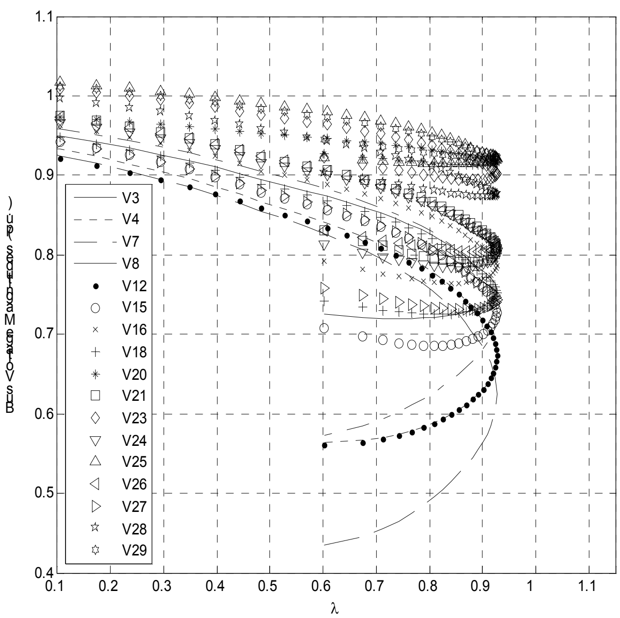

Figure 5: PV-Curves with Induction Motor Load Components as Indicated by Table 1.

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